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Stationarity and Unit Roots in Time Series: Theoretical Insights and Practical Considerations

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ABSTRACT

Time series data plays a central role in econometrics and finance, being the foundation of macroeconomic modelling, financial prediction, and risk assessment. Ensuring stationarity is, however, paramount, as non-stationary series may lead to erroneous statistical inference and spurious regressions. This essay discusses the problem of unit roots and stationarity, and gives a thorough description of different unit root tests. It starts with the old ones like the Dickey-Fuller test and Phillips-Perron (PP) test, and then proceeds to the newer ones, like the KPSS test, which is a stationarity test, not a test of unit roots. Structural tests such as Zivot-Andrews and panel unit root tests can also be employed. The research uses these tests to an empirical data set of macroeconomic variables and financial time series, assessing whether the series are stationary or need to be transformed. Empirical results underscore the significance of stationarity in Vector Autoregression (VAR), Cointegration Analysis, and forecasting models like ARIMA and GARCH. The findings indicate the relevance of proper unit root testing in economic and financial modelling to ensure firm policy choices and solid investment choices. The paper also explores methodological difficulties, such as small-sample bias, structural changes, and nonlinearity. Nonlinear unit root tests and machine learning strategies should be pursued in future work to enhance prediction accuracy.

Keywords: *Stationarity, Unit Root, VAR framework, GARCH, Panel Data, Time Series.*

JEL Classification: *C22, C32, E37, G17.*

I. Introduction

Time series data is measurements made over time, usually recorded at equal intervals like a day, month, or a year (Box et al., 1976; Jenkins & Box, 1976; Box et al., 2015). In contrast to cross-sectional data, which is a snapshot at a point, time series data is concerned with the development of variables and is therefore of high importance when analysing trends, cycles, and long-run relationships (Enders, 2008). Time series analysis has broad applications in econometrics in macroeconomic variables like interest rates, inflation, GDP, and unemployment to assist policymakers in evaluating economic stability and making proper decisions (Hamilton, 1994). In finance, it is crucial in asset pricing, volatility modelling, risk management, and portfolio optimization (Tsay, 2010). Analysts apply time series models to predict stock prices, exchange rates, and interest rates to enable investors to make smart financial decisions (Diebold, 2015).

The importance of time series data to finance and econometrics is once more underscored by the beginning of more cutting-edge statistical and econometric tools and techniques to estimate and analyse it. In addition, the conventional tools such as ARIMA models (Box et al., 1976; Jenkins & Box, 1976; Box et al., 2015). VAR models (Sims, 1980), and cointegration models (Engle & Granger, 1987) give the fundamental platform to investigate long-run equilibrium association and interdependency between economic variables. Furthermore, Bollerslev's

(1986) GARCH model is applied extensively in financial and commodity markets to capture volatility clustering, where periods of high and low volatility are likely to be persistent. Also advanced time-series models like state-space models and machine learning driven forecasting tools and techniques have been increasing prediction accuracy, particularly in high-frequency data analysis (Makridakis et al., 2019).

Moreover, time series is expected to addresses very vital econometric concerns such as seasonality, stationarity, and structural breaks in the data set, which, if not regulated, expected to yield misleading conclusions (Stock & Watson, 2007, 2011). Stationarily of the time-series data set, an important feature offering stable statistical properties over the study period, is analysed by tests such as the ADF Test (Augmented Dickey-Fuller) and the P-P test (Phillips-Perron). Yet another element called seasonality component, which is widespread in financial time series, is generally handled by using seasonal breakdowns methods such as SARIMA models (Hyndman & Athanasopoulos, 2018). On the other hand, structural breaks in the time series dataset, which are revealing of shifts in regimes, are very vital in understating the proposed model's accuracy and are generally investigated by applying tests such as Chow break-point test and Bai-Perron test (Bai-Perron, 2003).

With the growing complexity of international markets and access to enormous financial datasets, the marriage of conventional time series models with machine learning methods has become more prominent. Methods that blended econometric models with advanced deep learning methods like Long Short-Term Memory (LSTM) networks and recurrent neural networks (RNNs) have proven to be better performing in financial forecasting (Zhang et al., 2020). The use of time series econometrics in risk management, policy-making, and investment decision-making highlights its essentiality in current economic and financial studies.

Stationarity is very important in time series analysis to ensure that the statistical characteristics of a process mean, variance, and autocorrelation are constant over time. This constancy is basic to the usability of most statistical models, such as Autoregressive Integrated Moving Average (ARIMA), whose dependability is based on stationarity assumption while making predictions. Applying these models to non-stationary data gives erroneous results, as the changing statistical properties negate the model's fundamental assumptions. Also, stationarity increases predictability of a time series; when patterns underneath is stable, models can spot persistent trends and relationships, producing more accurate predictions. Also, most statistical methods and tests, including hypothesis testing and confidence intervals, rely on stationarity. Violations of this assumption can lead to false conclusions. Stationarity is therefore not only a technical requirement but a necessary prerequisite for valid and meaningful statistical inference in time series analysis.

Stationarity in time series data involves the characteristic wherein statistical properties like mean, variance, and autocovariance remain unchanged over a period of time. There exist various types of stationarity that have certain criteria to be satisfied. Strict stationarity, which is also termed as strong stationarity, asks for the fact that the joint distribution of any subset of the observations should remain the same whenever shifted in terms of time (Priestley, 1981). Weak stationarity (also referred to as covariance or second-order stationarity) is less strict in that only the first two moments autocovariance and mean are time-invariant (Hamilton, 1994). A time series is considered trend stationary if it contains a deterministic trend but returns to a constant mean upon the subtraction of the trend component (Stock & Watson, 2015). Conversely, difference stationarity is where a series that is stationary upon differencing can be said to have unit roots prior to being transformed (Dickey & Fuller, 1979). Still another type of stationarity includes seasonal stationarity, where there are periodic features but stability among corresponding periods exists (Box et al., 2015). To create stationarity, researchers routinely use statistical tests such as the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1981), the Phillips-Perron (PP) test (Phillips & Perron, 1988), and the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al., 1992). The tests check if a series is endowed with a unit root, representing non-stationarity, or if it contains stable features along the passage of time.

The unit root theory is central to time series analysis because it captures the existence of non-stationarity in a data set and affects model reliability and forecastability. The unit-root time series possesses a stochastic trend, in which shocks produce lasting effects instead of fading with time (Nelson & Plosser, 1982). The existence of unit root in the dataset can be detected by several tests such as ADF test, P-P test, and KPSS test. More advanced techniques have been created in recent years to increase test efficiency. The Ng-Perron test (Ng & Perron, 2001) is highly useful for short samples and offers greater test precision and stability, whereas the Elliott-Rothenberg-Stock (ERS) test (Elliott et al., 1992) possesses greater statistical power to detect near-unit root processes. The existence or otherwise of a unit root has grave implications for econometric modelling since popular regression techniques such as Ordinary Least Squares (OLS) produce spurious inferences if these are implemented in non-stationary data (Granger & Newbold, 1974). For addressing such a situation, methods like cointegration analysis (Engle & Granger, 1987) are used when non-stationary variables have a long-run stable relationship. Current research further develops unit root testing by including structural breaks (Bai & Perron, 2003; Zivot & Andrews, 2002), since omitting breaks can result in making wrong conclusions regarding stationarity. It is imperative to understand unit roots in macroeconomic and financial time series modelling because not factoring them can result in invalid economic policy suggestions and unsound predictive models (Stock & Watson, 2015).

The main objective of this paper is to give a full account of unit root tests, how they are applied, and why they matter in empirical data, especially against the backdrop of time series. The existence of a unit root in a time series is an indication of non-stationarity that can result in misleading conclusions as well as spurious results of regression if not corrected (Granger & Newbold, 1974). The various statistical tests for the identification of unit roots are discussed in this study, such as the Augmented Dickey-Fuller (ADF) test (Dickey & Fuller, 1981), Phillips-

Perron (PP) test (Phillips & Perron, 1988), and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test (Kwiatkowski et al., 1992), among others. Knowledge of whether or not a time series is stationary is important in selecting the right modelling methods, e.g., differencing to make the series stationary or cointegration testing to detect long-run links (Engle & Granger, 1987). Finally, unit root tests have also been improved in recent years with the introduction of the Elliott-Rothenberg-Stock (ERS) test (Elliott et al., 1992) and the Ng-Perron test (Ng & Perron, 2001), have enhanced the precision and consistency of stationarity tests, especially in small samples. Non-stationarity is addressed in many disciplines, such as economics, finance, and environmental science, since its improper treatment can skew forecasting models and policy choices (Stock & Watson, 2015). Through the systematic comparison of various unit root tests and their real-world implications, this research seeks to give researchers a concise guideline for addressing non-stationarity in time series data.

II. LITERATURE REVIEW

The unit root and stationarity test has continued to be central in time series analysis because of their fundamental implications on modelling and forecasting. A stationary time series is one whose statistical attributes such as mean and variance are constant over time. Conversely, the presence of a unit root indicates non-stationarity and means that shocks to the system can create long-lasting effects, rendering analysis difficult and can generate spurious regression results if not treated by an adequate correction. Dickey and Fuller's first work (1979) constructed the Dickey-Fuller (DF) test, employed to ascertain the null hypothesis of unit root against the alternative of stationarity. They generalized this to the Augmented Dickey-Fuller (ADF) test to consider higher-order autoregressive processes based on identification of limitations in handling more complex data structures. Later, Phillips and Perron (1988) developed the Phillips-Perron (PP) test, which provides a non-parametric method of unit root testing, overcoming serial correlation and heteroskedasticity problems without the necessity of augmenting lag terms. Unlike these tests, which take non-stationarity as the null hypothesis, the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test takes stationarity as the null, offering an alternative view in unit root testing. In response to the limitations of the conventional tests, especially in the case of small samples or near-unit root cases, more advanced techniques have been devised by researchers. The Elliott-Rothenberg-Stock (ERS) test provides a generalized least squares (GLS) detrending method to augment test power to identify unit roots. Likewise, the Ng-Perron test remedies size distortion and low power problems by offering alternative test statistics that are more reliable, particularly in small samples.

The empirical use of such tests is enormous, covering economics, finance, and environmental sciences, among many other fields. In econometrics, for example, it is essential to ascertain the stationarity of macroeconomic variables in order to make correct modelling and policy analysis. A survey of the popularity and use of unit root tests puts their central importance in appreciating the integration properties of time series data into perspective, an understanding that has important implications for policy-making as well as econometric modelling. There has also been a recent development that examined the Bayesian method of unit root and cointegration testing, providing an alternative in the form of a fully Bayesian significance test (FBST). The method presents a new way of hypothesis testing for time series, possibly providing a more detailed explanation of the existence of unit roots and cointegration relationships. Even with the emergence of numerous unit root tests, there are still issues, especially regarding the power and size properties of the tests in finite samples. Research on the usefulness and need for unit root tests in small samples indicates that the tests can have arbitrarily low power in finite samples, and therefore caution is needed when applying and interpreting them.

The research on stationarity and unit root tests has been central to time series analysis, determining the precision of econometric models and forecasts. The following is an overview of seminal and key contributions to this research:

Dickey and Fuller's Contributions: Dickey and Fuller (1979) developed the Dickey-Fuller (DF) test, which tests the null hypothesis of the presence of a unit root in an autoregressive time series model. The test paved the way for other approaches to dealing with non-stationarity in time series.

Phillips and Perron's Contributions: Phillips and Perron (1988) developed the Phillips-Perron (PP) test, a non-parametric unit root test. The PP test has increased robustness against serial correlation and heteroskedasticity without having to add leading lag terms, providing a competing method to the ADF test.

Kwiatkowski-Phillips-Schmidt-Shin (KPSS) Test: Kwiatkowski et al., (1992) presented the KPSS test, where the stationarity is assumed under the null hypothesis. It is an alternative approach to unit root testing and provides a means of verifying findings of tests such as ADF and PP.

Elliott-Rothenberg-Stock (ERS) Test: Elliott et al. (1992) created the ERS test, which uses a generalized least squares (GLS) detrending method to increase the power of unit root tests, particularly in the case of small samples or near-unit root.

Ng-Perron Test: Ng and Perron (2001) solved problems of size distortions and low power in unit root testing through the suggestion of new test statistics. Their method enhances the efficiency of unit root tests, especially in finite samples.

These early investigations have considerably strengthened the unit root detection methodologies as well as the stationarity test methods, improving the reliability of time series analysis in numerous fields. It is essential to determine stationarity in time series data to facilitate precise modelling and inference. A significant advance in this regard is the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test of Kwiatkowski et al. (1992). Unlike traditional unit root tests that postulate non-stationarity under the null hypothesis, the KPSS test assumes stationarity under the null, offering a contrasting perspective in time series analysis. Basing on the KPSS model, recent research has pushed its applicability to higher data structures. For example, the KPSS test has been generalized to functional time series so that it can test stationarity for infinite-dimensional cases. Furthermore, seasonal component considerations have resulted in the introduction of the KPSS test with seasonal dummies, making it more applicable for the case of the data containing periodic patterns. Its resistance to outliers has also been assessed in order to test its strength in different conditions of data. Concurrent with these advances, panel unit root tests have been introduced to accommodate datasets that have both cross-sectional and time series dimensions. Initial approaches, referred to as first-generation tests, assumed cross-sectional independence. This, however, is generally unrealistic in real-world applications where cross-sectional units can be interdependent. To overcome this, second-generation panel unit root tests have been introduced, specifically accounting for cross-sectional dependence. These developments improve the power and robustness of unit root testing in panel data settings.

Other improvements involve the inclusion of structural breaks in panel unit root testing methods. Acknowledging that actual data tend to undergo structural breaks, methods have been suggested to include such breaks, thus enhancing the robustness of stationarity tests. The use of Generalized Least Squares (GLS) detrending methods has also been investigated to make panel unit root tests more efficient, especially in the case of incidental trends.

Althogh there is wide literature available on investigating the existence of unit root in time series dataset, the major challenge is the most relevant test for investigating the existence of unit-root remains a difficult one for analysts and researcher to select. On the one-hand traditional methods such as ADF test, P-P test, are extensively applied in econometrics, however, they are prone to give misleading results owing to their vulnerability to structural breaks, instability in sample, and existence of serial correlation (Perron, 1989). On the other hand, to address these shortcomings, several other tests namely KPSS test and panel unit root tests (Levin et al., 2002; Im et al., 2003) have been widely used in literature. Therefore, it is still challenging for researchers to select a suitable test owing to their inbuilt variations in assumptions, power, and usability under different data situations. Consequently, a fundamental research need is to systematically direct researchers on the choice of an ideal stationarity test depending on the unique properties of their time series data. Structural breaks, cross-sectional dependence across panel datasets, and higher power requirements against near-unit root alternatives create a need for more advanced understanding of these tests (Zivot & Andrews, 1992; Bai & Perron, 2006). More recent tests, such as the Elliott-Rothenberg-Stock (ERS) test (Elliott et al., 1992) and the Ng-Perron test (Ng & Perron, 2001), seek to better detect stationarity by avoiding some of these issues, but a general test selection framework has yet to emerge. This research aims to fill this void by offering an organized summary of the different unit root tests, their theoretical basis, and their empirical significance. Through the analysis of the advantages and disadvantages of each test under different scenarios i.e., small sample properties, structural change, and cross-sectional dependence this research will act as a hands-on guide for applied econometricians and researchers in choosing the most suitable stationarity test. Filling this gap is crucial to the guarantee of the robustness of empirical results in time series modelling and the prevention of spurious regression caused by undetected non-stationarity (Granger & Newbold, 1974).

III. UNIT ROOT PROCESS

Unit Root Process: Mathematical Model and Implications The notion of a unit root process is basic in time series econometrics and describes a stochastic process wherein the present value of a series highly relies on past values of the series. A time series has a unit root if it describes a non-stationary process where permanent impact results from shocks to the series, creating unforeseeable long-run behaviour (Nelson & Plosser, 1982; Hamilton, 1994).

Mathematical Representation of a Unit Root Process: AR (1) Model Unit root process is a stochastic time series process in which the current value of a variable relies significantly upon past values so that the process is non-stationary. The most primitive type of a unit root process can be outlined using the first-order autoregressive (AR (1)) model, specified as:

$$y_t = \emptyset y_{t-1} + \varepsilon_t$$

Where, y_t is the dataset at time t, $\emptyset$ is the auto-regressive factor coefficient, and ε_t is the error term with $N(0, \sigma^2)$.

A time series is stationary if the statistical characteristics, like mean and variance, do not change with time. The condition is fulfilled when $|\phi|<1$ (Box & Jenkins, 1976; Fuller, 1976). Yet, if $\phi=1$, the process has a unit root, resulting in:

$$y_t = y_{t-1} + \varepsilon_t$$

which is a random walk (Dickey & Fuller, 1979). Under this scenario, every shock to the system is permanent, making the series diverge instead of returning to a mean (Phillips, 1987).

Higher-Order Unit Root Processes For a generalized autoregressive process of order ???? p (AR(p)), the model becomes:

$$y_t = \sum_{i=1}^{p} \emptyset_i y_{t-1} + \varepsilon_t$$

The existence of a unit root in an AR(p) procedure can be determine by the following equation:

$$1 - \sum_{i=1}^{p} \emptyset_i L^i = 0$$

where L is the lag operator, and a unit root is present if at least one root of this equation is on the unit circle (Hamilton, 1994). A time series with several unit roots is called an integrated process of order *d*, denoted as *I*(d), where differencing *d* times is needed to make it stationary (Granger & Newbold, 1974).

Implications of a Unit Root in Time Series Data

Having a unit root implies a number of deep consequences when analysing time series:

Non-Stationarity: A unit root process is non-reverting around its mean and has an expanding variance over time, resulting in an undefined long-run mean as well as an infinite variance (Stock & Watson, 1988).

Spurious Regression: Regression of two or more independent non-stationary series may yield deceptive results, including high R^2 values and significant t-statistics, when there is no real relationship between the variables (Granger & Newbold, 1974).

Persistence of Shocks: Any shock ϵ_t to a unit root process permanently affects the series, and long-run forecasting becomes unreliable (Phillips & Perron, 1988).

Unpredictable Trends: In contrast to stationary processes that return to a stable equilibrium, a unit root process traces an unpredictable path, rendering policy and economic forecasting highly uncertain (Nelson & Plosser, 1982).

The occurrence of a unit root in a time series has important theoretical as well as practical consequences. It influences the correctness of statistical inference, produces erroneous regression estimates, and undermines the accuracy of forecasts. Identification of unit root processes is important in empirical work to ensure correct modelling and economically significant interpretations.

Structural Breaks and Unit Roots

Traditional unit root tests, such as the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, assume that the data-generating process is stable over time. A structural break exists when there is a dramatic change in the underlying process that is producing a time series, causing changes in parameters like the mean, variance, or trend. Economic crises, policy shocks, or technological breakthroughs can structurally break processes and thus contaminate the results of unit root tests as well as lead to false inference (Perron, 1989). Tests that control for such breaks, including the Zivot-Andrews (1992) test and Perron (1997) test, are more accurate. A standard AR model with unit root is given as follows:

$$y_t = \rho y_{t-1} + \varepsilon_t$$

If ρ=1, the time series acted to follows a random path, signifying the existence of a unit root. When there exists a structural break at time T_b, the model takes the following form:

$$y_t = \mu_0 + \mu_1 D_t + p y_{t-1} + \beta_t + \gamma D_t t + \varepsilon_t$$

Where, D_t =1 if t>T_b (break period), otherwise D_t =0, μ_0 is the initial intercept. μ_1 captures the intercept change because of the break, β is the deterministic trend, and γ captures the trend change because of the break.

This specification enables the time series to undergo a level shift (intercept) as well as trend shift at time T_b

Perron's (1989) Structural Break Model

Perron (1989) proposed three major types of structural break models:

Additive Outlier Model (AO): Immediate Break

$$y_t = \alpha + \beta t + \theta D_t + \rho y_{t-1} + \epsilon_t$$

Innovational Outlier Model (IO):

$$y_t = \alpha + \beta t + \rho y_{t-1} + \gamma D_t + \epsilon_t$$

Break in Both Level and Trend.

$$y_t = \alpha + \beta t + \theta D_t + \gamma D_t t + \rho y_{t-1} + \epsilon_t$$

Perron's test modifies the ADF test to check for unit roots while accounting for structural breaks.

Zivot-Andrews (1992) Unit Root Test with Structural Breaks

In contrast to Perron's test (which takes a specified break date as given), the Zivot-Andrews (1992) test estimates the break date within the model. The model is expressed as:

$$y_t = \mu + \beta t + \theta D_t + \gamma D_t t + \rho y_{t-1} + \sum_{j=1}^{k} c_j \Delta y_{t-j} + \epsilon_t$$

D_t is a dummy variable indicating the break date. If = 1 ρ=1, the series has a unit root.

Structural breaks may result in misclassification of stationarity in unit root tests to produce spurious outcomes. Perron (1989) and Zivot-Andrews (1992) developed augmented unit root tests that correct for breaks in intercept and or trend. It is crucial to know these models to obtain precise econometric modelling and forecasting.

Different types of tests available to check the unit root

Unit root tests are statistical techniques used to check if a time series is stationary or not. Unit root tests aid in checking if a shock to a time series has a temporary or permanent influence, and it is essential in correct modelling and forecasting. Unit root tests can be categorized into classical tests, stationarity-based tests, tests for structural breaks, and panel unit root tests depending on various assumptions and data structure. The selection of the proper test is needed to provide solid econometric analysis and valid inference in time series studies.

IV. ASSESSING STATIONARITY ISSUES

THE DICKEY-FULLER (DF) TEST AND THE AUGMENTED DICKEY-FULLER (ADF)

The Dickey-Fuller (DF) test is a statistical test that can be employed to establish whether a time series has a unit root, thus indicating non-stationarity (Dickey & Fuller, 1979). The test relies on the autoregressive process of order one, often written as:

$$Y_t = \rho Y_{t-1} + \varepsilon_t$$

where Y_t is the time series, ρ is the coefficient, and ϵ_t is a white noise error term. The main aim of the DF test is to investigate whether ρ=1, which would signify a unit root and hence non-stationarity. The null hypothesis to be tested is:

Null Hypothesis (H0): The series is unit-rooted (ρ=1), i.e., the time series is non-stationary. Alternative Hypothesis (H1): The series is stationary (ρ<1).

To make testing easier, the equation is reformulated by subtracting Y_{t-1} from both sides:

$$Y_t - Y_{t-1} = (\rho - 1)Y_{t-1} + \varepsilon_t$$

The $\Delta Y_t = Y_t - Y_{t-1,}$ the equation can be re-written as follows:

$$\Delta Y_t = (\rho - 1)Y_{t-1} + \varepsilon_t$$

Let γ=ρ-1, the simple equation is as follows:

$$\Delta Y_t = \gamma Y_{t-1} + \varepsilon_t$$

The test is expected to checks whether γ=0 (ρ=1) indicating that there is a unit root in the dataset. However, if γ<0, indicating that the dataset is stationary, and if γ=0, indicating that the dataset is non-stationary. In case α intercept and βt trend component is included; the generalised equation is as follows:

$$\Delta Y_t = \alpha + \beta t + \gamma Y_{t-1} + \varepsilon_t$$

where δ=ρ−1. Testing for a unit root then simplifies to checking if δ=0. The test statistic of δ is compared with critical values given by Dickey and Fuller (1979), which have a non-standard distribution under the null hypothesis.

The Augmented Dickey-Fuller (ADF) test is an extension of the DF test with the inclusion of lagged differences to allow for higher-order autocorrelation in the error term (Dickey & Fuller, 1981). The Dickey-Fuller (DF) test is a basic unit root test that tests whether a time series has a unit root, which means non-stationarity. The Augmented Dickey-Fuller (ADF) test is an extension of the DF test that controls for higher-order autocorrelation in the time series by including lagged differences of the dependent variable. The ADF test equation is expressed as:

$$\Delta y_t = \delta y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-j} + \epsilon_t$$

In case α intercept and βt trend component is included; the generalised equation is as follows:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta y_{t-j} + \epsilon_t$$

Where, Yt = the time series variable $\Delta Y_t = Y_t - Y_{t-1}$ (first difference of Y_t), α = constant (drift term), βt = deterministic trend component γY_{t-1} = lagged level of Y_t, used to test for a unit root $\sum_{i=1}^{p} \alpha_i \Delta y_{t-j}$ = sum of lagged differences to account for serial correlation p = number of lagged difference terms ϵ_t = white noise error term

The use of the lag terms guarantees that the error term ϵ_t is white noise of the proposed equation. The DF and ADF tests are extensively employed in time series econometrics to check the stationarity of economic and financial data prior to using other modelling methods like cointegration analysis and vector autoregression (Granger & Newbold, 1974; Enders, 2008). The outcome of the tests has a significant implication in choosing correct transformations, including first differencing, to transform the series to stationarity in order to escape problems like spurious regression (Gujarati, 2002; Gujarati & Porter, 2009). The DF test is a critical tool in time series econometrics that enables researchers to identify non-stationarity and determine if differencing is required prior to using models such as ARIMA or VAR. Nonetheless, because of constraints such as residual autocorrelation, the Augmented Dickey-Fuller (ADF) test is generally used in real-world applications.

On the ADF test, the rejection of the null hypothesis of unit root depends on critical values. These values come from Monte Carlo simulations (Dickey & Fuller, 1979; MacKinnon, 1996) that are different from the normal t-distributions and depend on which model is assumed (no constant, constant added, or constant and trend). If the test statistic from ADF is smaller than the corresponding critical value in a more negative direction, reject the null hypothesis, suggesting stationarity. Otherwise, the series is said to be non-stationary. MacKinnon (1996) gave better finite-sample critical values, which are widely available in statistical software.

The Augmented Dickey-Fuller (ADF) test is commonly employed to detect unit roots in time series data; however, it has several drawbacks. The following are the main criticism against the test:

Low Power Against Near-Unit Root Alternatives: The ADF test tends to fail to reject the null hypothesis when the series is stationary but with a root near one (DeJong et al., 1992). This renders it less powerful in discriminating between genuine non-stationarity and near-stationary processes.

Sensitivity to Lag Length Choice: A proper number of lags in the augmented term needs to be selected for conducting the test. If the lags are in short supply, there can still be autocorrelation in the residuals, hence biased results. Excessive lags, in contrast, cut down on the power of the test (Ng & Perron, 2001).

Poor Performance in Small Samples: The ADF test tends to suffer from size distortions and poor power under small sample sizes, rendering it unsuitable for short time series data (Shiller & Perron, 1985).

Assumes No Structural Breaks: The ADF test does not adjust for structural breaks in the data, and if there is a structural break but one fails to control for it, the test can conclude that a unit root exists when in fact it does not (Perron, 1986, 1988).

Skewed Towards Non-Rejection of Null Hypothesis: The ADF test over-accepts the null hypothesis (i.e., existence of a unit root) even for a stationary time series, particularly when dealing with strongly persistent data (Elliott et al., 1992).

Reliance on Deterministic Terms: The result of the test depends on whether a trend or an intercept is specified in the model. Mis-specification of deterministic terms can result in deceptive outcomes (Harris & Sollis, 2003).

Because of these limitations, researchers resort to supplementing the ADF test with other tests such as the Phillips-Perron (PP) test (Phillips & Perron, 1988), the KPSS test, or panel unit root tests in the case of a series of time series.

PHILLIPS-PERRON TEST

The second test available to investigate the unit root in time series data is Phillips-Perron test also known as PP test (Phillips & Perron, 1988). This is a non-parametric test. Although it looks like Dickey-Fuller test but differs in its approach to addressing auto-correlation and heteroskedasticity of the residuals.

Although the Augmented Dickey-Fuller (ADF) test adds extra lagged difference terms to eliminate serial correlation, the PP test adjusts test statistics directly using a non-parametric correction derived from Newey-West estimators, which renders it immune to overall forms of autocorrelation and heteroskedasticity. Hypothesis Formulation: The PP test tests the null hypothesis that the time series has a unit root, against the alternative that it is stationary:

Null Hypothesis (H_0): The series is non-stationary (i.e., it has a unit root) ($\rho=1$).

Alternative Hypothesis (H_1): The series is stationary (no unit root present) ($\rho<1$).

The test statistic is calculated under the OLS estimate of ρ, but the standard error is corrected for autocorrelation and heteroskedasticity by non-parametric methods. The two most common test statistics employed in the PP test are:

The following is the mathematical equation of P-P test

$$y_t = \alpha + \beta t + \rho y_{t-1} + \epsilon_t$$

where: Y_t represents the time series, α is the intercept (optional), βt is an optional deterministic time trend, ρ is the autoregressive coefficient, and ϵt is a white noise error term. The main interest is in testing whether $\rho=1$. The PP test corrects the t-statistic from the ordinary least squares (OLS) regression of the above equation for autocorrelation and heteroskedasticity using the following modified test statistic:

$$Z_{\alpha} = T_{\hat{p}} - \frac{1}{2}\lambda\hat{S}^2$$

Where $T_{\hat{p}}$ is the standard t-statistic from the OLS regression, and $\lambda\hat{S}^2$is a correction factor for heteroskedasticity and serial correlation.

$$Z_{\alpha} = T(\hat{\rho} - 1) - \frac{1}{2}\lambda\hat{S}^2$$

where ρ is the regression estimated coefficient, T is the number of observations, i.e., the sample size, λ is a consistent estimator of the long-run variance, and $\hat{S}^2$is the estimated variance of the regression residuals. The Z-statistics are then compared with critical values from the Dickey-Fuller distribution to ascertain if the null hypothesis can be rejected. The PP test's critical values are from the Dickey-Fuller distribution, as in the ADF test.

Principal differences from the ADF Test

The ADF test adjusts for autocorrelation by including lagged difference terms (ΔY_{t-1}, ΔY_{t-2}, ...), while the PP test adjusts the standard errors non-parametrically. The PP test is less sensitive to heteroskedasticity and does not need lag selection. Nevertheless, the PP test is also prone to size distortions in finite samples, such that it could reject the true null hypothesis too frequently (Ng & Perron, 2001).

Advantages of the PP Test

Strong to Autocorrelation and Heteroskedasticity: Unlike ADF, PP test does not need lag length selection because it adjusts for serial correlation non-parametrically (Phillips & Perron, 1988). Flexibility: It is possible to use it for time series with structural changes without specifically including extra terms in the model (Perron, 1986).

Limitations

In spite of these benefits, the PP test has been faulted for size distortions and for being too sensitive to structural breaks, which increases the likelihood of false inference in finite samples (Ng & Perron, 2001). To address these problems, other tests such as the Elliott-Rothenberg-Stock (ERS) test and the Ng-Perron test have been proposed to improve power and size properties (Elliott et al., 1992).

KWIATKOWSKI-PHILLIPS-SCHMIDT-SHIN TEST

Kwiatkowski-Phillips-Schmidt-Shin Test is also popularly known as KPSS test, introduced by Kwiatkowski, Phillips, Schmidt, and Shin (1992). The test has been introduced to capture the stationarity in the time series data, as opposed to the ADF test or PP tests, which are introduced compute unit root in dataset. However, the KPSS test assumes that the dataset is stationarity under null hypothesis whether the data series has deviated away from stationarity due to existence of unit root.

Mathematical Formulation

The KPSS test decomposes a time series Y_t into three components: a deterministic trend T_t, a random walk R_t, and a stationary error term ϵ_t:

$$y_t = T_t + R_t + \epsilon_t$$

where: $T_t = \beta t$ is a deterministic trend component, R_t is a random walk component in the equation.

$$R_t = R_{t-1} + u_t$$

With $u_t \sim IID(0, \sigma_u^2)$, and ϵ_t is a stationary error term.

The null and alternative hypotheses for the KPSS test are:

Null Hypothesis (H_0): The time series is stationary (i.e., $\sigma_u^2 = 0$, which implies R_t is constant). Alternative Hypothesis (H_1): The time series is non-stationary due to the presence of a unit root (i.e., $\sigma_u^2 > 0$, meaning R_t follows a stochastic trend).

The KPSS test statistic is based on the LM statistic:

$$KPSS = \frac{\sum_{t=1}^{T} S_t^2}{T^2 \hat{\sigma}^2}$$

where: St is the partial sum of residuals from the regression of Yt on a deterministic component (constant or trend), $\hat{\sigma}^2$ is the long-run variance of the residuals, T is the sample size.

The critical values are provided by Kwiatkowski et al. (1992) based on Monte Carlo simulations.

Suitability of the KPSS Test

The KPSS test is applicable in situations where researchers need to verify stationarity instead of testing for unit roots. It is usually combined with the ADF or PP tests.

If: ADF/PP rejects the unit root null, and KPSS does not reject stationarity → Strong evidence of stationarity.

ADF/PP does not reject the unit root null, and KPSS rejects stationarity → Strong evidence of non-stationarity.

ADF/PP and KPSS produce contradictory conclusions → Inconclusive, as additional research may be needed (e.g., structural breaks).

KPSS test has some merits that render it a valuable instrument for time series analysis. Unlike the traditional unit root tests, such as the Augmented Dickey-Fuller (ADF) test and Phillips-Perron (PP) test, used to test for the existence of a unit root, the KPSS test tests for stationarity of a series and presents a different perspective (Kwiatkowski et al., 1992). This renders it particularly beneficial when testing for stationarity assumptions before implementing models such as ARIMA or VAR, in which stationarity is an essential assumption. KPSS test is also often used in combination with other unit root tests to increase the robustness of conclusions if both tests agree with their conclusions, more reliable inferences about stationarity can be made (Schwert, 1989).

In spite of its merits, the KPSS test has some shortcomings. One of its main weaknesses is that it is sensitive to the bandwidth selection in the estimation of long-run variance, and this can impact the results of the test greatly (Schwert, 1989). In addition, the test is low power in identifying trend stationarity and difference stationarity, so it is not as reliable under some conditions (Glynn et al., 2007; Stock, 1994; Byrne & Perman, 2006). The second issue is that the KPSS test is subject to size distortions, particularly for small samples, which can cause wrong rejections of stationarity even if the series is stationary (Sul et al., 2005). Because of these issues, it is recommended by researchers that the KPSS test be used in conjunction with unit root tests to arrive at more definitive conclusions about stationarity in time series data (Granger, C. W. J. (2001).

THE ZIVOT-ANDREWS TEST

The Zivot-Andrews (ZA) Test is a modification of standard unit root tests that is sensitive to structural breaks in a time series. Traditional tests such as the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests rely on the assumption that the underlying data have a stable process over time, which might not hold in practice where economic or financial time series tend to suffer from structural changes because of policy changes, financial crises, or technological innovations (Zivot & Andrews, 1992). The ZA test expands unit root testing by permitting a single structural break that is endogenous, enhancing precision in identifying stationarity with regime shifts.

The Zivot-Andrews test is especially designed to examine time series data which undergo unprecedented structural breaks, to which standard unit root tests fail to adapt. Such breaks may arise as a result of intense economic crises, for example, the 2008 Global Financial Crisis that witnessed an extreme degree of turmoil in the financial market and long-run macroeconomic tendencies. Additionally, policy shocks, including money policy or interest rates, can introduce sudden economic variable changes and thus the necessity of the test for stationarity that accounts for structural breaks. Natural catastrophes, like pandemics caused by COVID-19, also introduce unforeseen volatility of economic, finance, and health time series. Furthermore, structural reforms or deregulations such as trade liberalization policies can radically transform market dynamics, which requires a more sophisticated method like the Zivot-Andrews's test to establish stationarity properly. By considering a single endogenously determined structural break, this test enhances the precision of unit root identification, and hence it is a useful method for empirical analyses in macroeconomics and finance (Zivot & Andrews, 1992; Perron, 1989; Narayan & Popp, 2010).

Conventional unit root tests usually spuriously suggest non-stationarity in the presence of a structural break and thus make ZA test an indispensable alternative tool for empirical applications in macroeconomic and financial data (Perron, 1988). Hypothesis for the Zivot-Andrews Test The test adopts the conventional null hypothesis (H_0) and alternative hypothesis (H_1) structure:

H_0 (Null Hypothesis): The time series has a unit root (i.e., it is non-stationary, even after a structural break).

H_1 (Alternative Hypothesis): The time series is stationary once a one-off structural break is controlled for.

Mathematical Formulations of the Zivot-Andrews Test

The ZA test generalizes the Augmented Dickey-Fuller (ADF) equation by introducing a single break point (TB) in intercept, trend, or both. The overall model is:

$$y_t = \mu + \theta DU_t + \beta t + \gamma DT_t + \alpha y_{t-1} + \sum_{j=1}^{k} \delta_j \Delta y_{t-j} + \varepsilon_t$$

where: DU_t =1 for t>TB, otherwise 0 (captures a break in the intercept), DT_t =(t−TB) for t>TB, otherwise 0 0 (captures a break in the trend), y_t is the time series variable α is the unit root coefficient, k is the lag order to account for autocorrelation, and ϵ_t is the error term The break date TB is determined endogenously by the test, i.e., it is chosen depending on where the structural change reduces the t-statistic for testing α=1.

Break Specification

The dummy variable D_t is different depending on the nature of break under consideration: Intercept Shift (Model A): D_t =1 if t >T_B, else 0.

Trend Shift (Model B): D_t = t−T_B if t>T_B , else 0.

Both Intercept & Trend Shift (Model C): A combination of above two.

The null hypothesis (H0) is that the series contains a unit root with a structural break (γ=0), while the alternative hypothesis (H1) proposes stationarity with a break (γ<0). The break point T_B is chosen endogenously as the point at which the t-statistic for γ is smallest.

This technique has the advantage of overcoming standard unit root tests in that it can handle one structural break but is not suited for multiple breaks, which reduces its utility when working with long-term economic and financial series.

Advantages of ZA test

The Zivot-Andrews's test is a significant advancement on traditional unit root tests like the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, particularly in the presence of structural breaks. Its most significant strength lies in the fact that it can account for a single endogenous break, reducing the likelihood of rejecting a time series as non-stationary while in reality the shift is caused by an external shock (Zivot & Andrews, 1992). This renders the test particularly useful in economic and finance research, wherein macroeconomic figures frequently change instantaneously owing to policy change, financial shocks, or other kinds of shocks. By acknowledging the breaks, thereby incorporating them within the framework used for testing, the accuracy for determining stationarity is significantly increased (Perron, 1989). Another major benefit is that the test endogenously sets the breakpoints in the dataset as opposed to expecting them to be pre-specified, enhancing its flexibility and generalizability across different empirical analyses (Bai & Perron, 2003).

Disadvantages of ZA test

Although it has its merits, the Zivot-Andrews (ZA) test has some limitations that need to be taken into consideration by researchers. One of its main disadvantages is that it can only accommodate a single structural break and is therefore less appropriate for long-run time series data that might undergo multiple breaks as a result of repeated economic shocks or policy reforms (Lumsdaine & Papell, 1997). Also, the test has lower power in small samples, so it can fail to yield definitive results when the sample size is small (Perron, 1997). Its sensitivity to break date estimation is another important limitation—the stationarity test's reliability can be affected if the estimated break is mis-estimated, resulting in spurious inferences (Narayan & Popp, 2010). These issues underscore the importance of researchers' judicious interpretations of ZA test results, particularly when working with time series data with possible multiple structural breaks or small sample restrictions.

PANEL UNIT ROOT TEST

Pane Unit Root Test has two different versions are available one is LLC (Levin-Lin-Chu) and the other is IPS (Im-Pesaran-Shin) Panel unit root tests are an extension of traditional time series unit root tests to investigate stationarity in the collected panel data, where various cross-sectional elements such as sectors, firms, nations etc. are observed over a period. Panel unit root tests have been extensively applied in empirical studies with macroeconomic and financial panel data. They are especially suitable for datasets that include: Macroeconomic variables like GDP, inflation, and interest rates for different countries (Maddala & Wu, 1999). Financial market data like stock prices and exchange rates for different firms or regions (Pesaran, 2007). Industry-level datasets where firms can have varying stationarity characteristics but have similar economic trends (Baltagi & Baltagi, 2008). The LLC test is more appropriate for balanced panels with homogeneous stationarity, whereas the IPS test is better for heterogeneous panels where unit root processes vary across units (Breitung J., & Pesaran,

M. H, 2005, 2008). These tests provide greater statistical power by leveraging cross-sectional information and have a wide application base in macroeconomics, finance, and social sciences (Baltagi & Baltagi, 2008).

LEVIN-LIN-CHU (LLC) TEST.

The tests yield higher statistical power by exploiting cross-sectional information and have a broad application base in macroeconomics, finance, and social sciences (Baltagi & Baltagi, 2008). Levin-Lin-Chu (LLC) Test. The Levin-Lin-Chu (LLC) test (Levin, Lin, & Chu, 2002) is one of the first panel unit root tests, imposing a common unit root process across the cross-sections. It relies on the following augmented Dickey-Fuller equation:

$$\Delta y_{it} = \alpha_i + \rho y_{i,t-1} + \sum_{j=1}^{p} \beta_j \Delta y_{i,t-j} + \varepsilon_{it}$$

where: y_{it} it is the time series for unit *i* at time t, α_i represents individual fixed effects, ρ is the coefficient of interest testing for a unit root, β_j captures lagged differences to correct autocorrelation, ϵ_{it} is the error term.

Hypotheses Null Hypothesis (H_0): All series contain a unit root ($\rho=0$).

Alternative Hypothesis (H_1): All series are stationary ($\rho<0$).

As LLC assumes a homogenous unit root (i.e., all cross-sections have the same stationarity properties), it is convenient for panel datasets that are very similar to each other but can be too constraining for heterogeneous panels (Breitung & Pesaran, 2005, 2008).

Advantages and Disadvantages of the LLC Test

The LLC test has various significant benefits, which render it significant in panel data analysis. One such benefit is enhanced statistical power, as aggregation of information over stations enhances test effectiveness (Levin et al., 2002). It is also particularly suitable for balanced panels with constant stationarity properties, as it is the best test for datasets where all stations have similar trends (Maddala & Wu, 1999). But a serious limitation of the LLC test is its stringent common unit root assumption, which could be violated in actual data sets where various cross-sections display different stochastic behaviours (Pesaran, 2007). Also, the test is not very accommodating to heterogeneity, thus restricting its usage to panel structures with variance homogeneity (Breitung & Pesaran, 2005, 2008).

THE IM-PESARAN-SHIN TEST

The Im-Pesaran-Shin (IPS) test (Im, Pesaran, & Shin, 2003) is a popular panel unit root test *that* permits heterogeneous unit root processes between cross-sectional units, overcoming the limiting assumption of homogeneity present in tests such as the Levin-Lin-Chu (LLC) test. As opposed to LLC, which makes a common unit root process assumption in the data, the IPS test acknowledges that time series for individual units in a panel could have varying stationarity characteristics, thereby more suited to work in real-world data, especially macroeconomic and financial datasets (Maddala & Wu, 1999). IPS is founded on an Augmented Dickey-Fuller (ADF) type of regression, which is adapted to take cross-sectional heterogeneity into account. The IPS standard regression equation is expressed as:

$$\Delta y_{it} = \alpha_i + \rho_i y_{i,t-1} + \sum_{j=1}^{p} \beta_{ij} \Delta y_{i,t-j} + \varepsilon_{it}$$

where Y_{it} is the variable of interest for each cross-section *i* at time *t*, α_i are the individual fixed effects, ρ_i is the coefficient of interest that is not constant across cross-sections, and ϵ_{it} is the error term (Im et al., 2003). In contrast to the LLC test, which pools the data to estimate a common value of ρ, the IPS test implements individual ADF regressions for every cross-sectional unit and then calculates the average of the individual t-statistics to obtain a panel-based test statistic (Pesaran, 2007). This improves the test's suitability to various panel datasets where cross-sectional units could exhibit different stochastic trends. Given the IPS test uses averages of cross-section ADF test statistics for each unit in a panel, it offers more flexibility than LLC, especially for scenarios where unit-level stationarity varies (Pesaran, 2007).

Advantages and Disadvantages of the IPS Test

One of the greatest strengths of the IPS test is that it can identify partial stationarity, in that it does not expect all cross-sections to be stationary at the same time. Rather, the alternative hypothesis is one where some of the cross-sections are stationery and others are not, and so it is more suitable for panels with heterogeneous integration orders (Breitung J, 2001; Breitung J., & Pesaran, M. H, 2005, 2008). The null hypothesis of the test (H_0) is that all series have a unit root ($\rho_i = 0$ for all $_i$), while the alternative hypothesis (H_1) is that at least some cross-sectional units are stationary ($\rho_i < 0$ for some *i*) (Im et al., 2003).

Because the IPS test accommodates individual unit root dynamics, it is more suitable for heterogeneous panels, like samples of several countries, industries, or firms which could react differently to economic shocks (Choi, 2001). Nevertheless, a drawback of the IPS test is that it is sensitive to cross-sectional dependence, and in the presence of strong correlations between units that are not controlled for, it may produce biased outcomes (Pesaran, 2007). In spite of this, the IPS test is one of the most popular panel unit root tests in econometric studies because it is flexible and resilient in dealing with various panel configurations (Baltagi & Baltagi, 2008). But one of its disadvantages is that it is not as strong as LLC when stationarity is uniformly homogeneous for panel units (Breitung J., & Pesaran, M. H, 2005, 2008). Moreover, the IPS test is also sensitive to cross-sectional dependence, i.e., omission of correlations between panel units can provide spurious results (Choi, 2001).

V. EMPIRICAL ANALYSIS AND REPORTING OF TEST RESULTS

While running the DF and ADF tests, it is very vital to decide on the correct model specification of the test equation. However, the decision is generally based on the nature of the time series dataset. If the series is trending around a nonzero mean but not a trend, the test must have an intercept (constant) only. If the series has an evident increasing or decreasing trend over time, then a trend-only specification is appropriate. If the series has both a nonzero mean and a trend, then the test equation should have both an intercept and a trend.

In situations where the series is strictly a random walk with no pattern of a systematic sort, the test may be performed with no intercept or trend (pure unit root case) (Dickey & Fuller, 1979; Said & Dickey, 1984). The use of a trend component is especially desirable, as it allows one to determine between a deterministic trend (trend-stationary process) and a stochastic trend (unit root process). Missing an essential component might cause the drawing of wrong inferences, i.e., spuriously assuming non-stationarity while the series is trend-stationary (Enders, 2008). The general strategy in applied research is to begin with the most general form, i.e., with intercept and trend included. If, on testing for statistical significance, the trend term turns out not to be statistically significant, re-estimation of the model using an intercept term only is performed. If the intercept is also not significant, the final test without both components is run to determine if there exists a pure unit root process (Greene, 2012). The rigorous methodology ensures that stationarity attributes of the series are properly identified so that incorrect conclusions are minimized.

ADF TEST RESULTS

TABLE1: DF/ADF TEST RESULTS

Test	Test Stats	1% CV	5% CV	10% CV
DF test	-2.445	-3.51	-2.9	-2.57
ADF with one lag	-3.13	-3.51	-2.9	-2.57
ADF with two lags	-2.74	-3.51	-2.9	-2.57

Note: CV is critical values (based on tau distribution, therefore, avoid taking decision only on p values)

Decision: DF test: Fail to reject H0, as the test statistics -2.445, which is greater than all critical values at 1% -3.51, at 5% -2.9, and at 10% -2.57, therefore, we cannot reject the null hypothesis (H0: the data is non-stationary).

ADF with one lag: Fail to accept H0, as the test statistics -3.13, which is lesser than critical value at 5% -2.9, and at 10% -2.57, therefore, we can reject the null hypothesis (H1: the data is stationary).

ADF with two lags: Fail to reject H0, as the test statistics -2.74, which is greater than all critical values at 1% -3.51, and at 5% -2.9, (ignored at 10% CV) therefore we cannot reject the null hypothesis (H0: the data is non-stationary).

THE PHILLIPS-PERRON TEST

The Phillips-Perron test was run to test stationarity of the time series data to investigate the existence of unit root in the dataset. Results are reported in Table 2.

TABLE2: PHILLPS-PERRON TEST RESULTS

Test Results	Test Stats	1% CV	5% CV	10% CV
PP at level and intercept	-2.36	-3.49	-2.9	-2.59
PP at level and intercept & trend component	-3.15	-4.05	-3.46	-3.16
PP First differenced (Intercept)	-4.8	-3.49	-2.9	-2.59
PP First differenced (Intercept & Trend)	-5.17	-4.05	-3.46	-3.16

Note: CV is critical values (based on tau distribution, therefore, avoid taking decision only on p values)

PP at level and intercept: The P-P test stats for the first model specification is -2.36 which is greater than all the critical values at 1% -3.49, at 5% -2.9, and at 10% -2.57, therefore we cannot reject the null hypothesis (H0: the data is non-stationary).

PP at level and intercept & trend component: The P-P test stats for the first model specification is -3.15 which is greater than all the critical values at 1% -4.05, at 5% -3.46, and at 10% -3.16, therefore we cannot reject the null hypothesis (H0: the data is non-stationary).

PP First differenced (Intercept): The P-P test stats for the first model specification is -4.18 which is lesser than all the critical values at 1% -3.49, at 5% -2.9, and at 10% -2.57, therefore we can reject the null hypothesis (H0: the data is stationary).

PP First differenced (Intercept & Trend): The P-P test stats for the first model specification is -5.17 which is lesser than all the critical values at 1% -4.05, at 5% -3.46, and at 10% -3.16, therefore we cannot reject the null hypothesis (H0: the data is non-stationary).

KPSS TEST

A Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test was done to test stationarity of the time series data. Unlike in the case of ADF and PP tests that test for unit root (non-stationarity), the KPSS test checks for stationarity under the null hypothesis. Results are reported in Table 3.

TABLE 3: KPSS TEST RESULTS

Test Results	Test Stats	1% CV	5% CV	10% CV
Test Results (KPSS) with intercept	0.451	0.740	0.464	0.347
Test Results (KPSS) with intercept & Trend	0.145	0.215	0.147	0.119
Test Results (KPSS) with intercept	0.351	0.740	0.464	0.348
Test Results (KPSS) with intercept & Trend	0.191	0.215	0.147	0.118
Test Results (KPSS) with intercept	0.881	0.740	0.464	0.347
Test Results (KPSS) with intercept & Trend	0.106	0.215	0.147	0.119

The test has been conducted to examine the following set of hypotheses and the inference is drawn based on the comparison of test statistic with the 1%, 5%, and 10% critical values.

H0: the dataset is stationary.

H1: The dataset is non-stationary.

Decision Rule under KPSS test: If the test statistic is greater than the critical value, reject H0 (Null) → The series is non-stationary. However, if the computed test stats ≥ the critical value, fail to reject the null hypothesis (H0) → The series is stationary.

Intercept Only: The reported test statistic is (0.451) < 1% critical value (0.740) – Fail to reject H0. The reported test statistic is (0.451) < 5% critical value (0.464) – Fail to reject H0. The reported test statistic is (0.451) > 10% critical value (0.347) –Reject H0. Hence, we accept null hypothesis at the 1%, 5% level, which means the series is stationary.

Intercept & Trend: The reported test statistic is (0.145) < 1% critical value (0.215) – Fail to reject H0. The reported test statistic is (0.145) < 5% critical value (0.147) – Fail to reject H0. The reported test statistic is (0.145) > 10% critical value (0.119) –Reject H0. Hence, we accept null hypothesis at the 1%, 5% level, which means the series is stationary when trend is included.

Intercept Only: The reported test statistic is (0.351) < 1% critical value (0.740) – Fail to reject H0. The reported test statistic is (0.351) < 5% critical value (0.463) – Fail to reject H0. The reported test statistic is (0.351) > 10% critical value (0.347) –Reject H0. Hence, we accept null hypothesis at the 1%, 5% level, which means the series is stationary.

Intercept & Trend: The reported test statistic is (0.191) < 1% critical value (0.215) – Fail to reject H0. The reported test statistic is (0.191) > 5% critical value (0.147) – Reject H0. The reported test statistic is (0.191) > 10% critical value (0.119) –Reject H0. Hence, we fail to accept null hypothesis at the 5% and 10% level, even though it is less than 1% CV, but it is rejected at 5% level, which means the series is non-stationary.

Intercept Only: The reported test statistic is (0.881) > 1% critical value (0.740) – Reject H0. The reported test statistic is (0.881) > 5% critical value (0.463) – Reject H0. The reported test statistic is (0.881) > 10% critical value (0.347) –Reject H0. Hence, we fail to accept null hypothesis at the 1%, 5% and 10% level, which means the series is non-stationary.

Intercept & Trend: The reported test statistic is (0.106) < 1% critical value (0.215) – Fail to reject H0. The reported test statistic is (0.106) < 5% critical value (0.147) – Fail to reject H0. The reported test statistic is (0.106) > 10% critical value (0.119) – Fail to reject H0. Hence, we accept null hypothesis at the 1%, 5%, and 10% level, which means the series is stationary.

ZIVOT-ANDREWS TEST

The Zivot-Andrews (ZA) test was performed to test for the existence of a unit root with a single structural break in the series. Unlike the conventional unit root tests that assume no breaks, the ZA test permits a break in the intercept, trend, or both and is therefore more appropriate for time series data that can have abrupt changes. The test adheres to the null hypothesis

(H0) the series has a unit root despite a structural break

(H1) the series is stationary with a structural break.

The choice depends on the comparison of the test statistic against the critical values at various significance levels (1%, 5%, and 10%). Results of the ZA test are shown in the table below:

Decision Rule: If Test Statistic < Critical Value, reject H0 → Stationary with a break. If Test Statistic ≥ Critical Value, fail to reject H0 → non-stationary, even with a break.

TABLE No. 4: SHOWING THE ZIVOT-ANDREWS (ZA) TEST RESULTS

Example	Brek Type	Test Stats	1% CV	5% CV	10% CV
1	Intercept	-4.96	-5.35	-4.81	-4.59
2	Trend	-5.51	-5.58	-5.09	-4.83
3	Intercept & Trend	-5.76	-5.58	-5.09	-4.83

In Example 1 (Intercept Only), the Zivot-Andrews (ZA) test statistic is -4.95. This statistic is compared to the critical values at various levels of significance: -5.34 (1%), -4.80 (5%), and -4.58 (10%). Since -4.95 is larger than the 1% critical value (-5.34), we cannot reject the null hypothesis (H0) at the 1% level. But -4.95 is below the 5% (-4.80) and 10% (-4.58) critical values, so we reject H0 at the 5% and 10% levels. From this outcome, we conclude that the series is stationary at the 5% and 10% significance levels but non-stationary at the 1% level despite having a structural break in the intercept.

In Example 2 (Trend Only), the Zivot-Andrews (ZA) test statistic is -5.51. This statistic is compared to the critical values at various levels of significance: -5.58 (1%), -5.09 (5%), and -4.83 (10%). Since -5.51 is larger than the 1% critical value (-5.58), we cannot reject the null hypothesis (H0) at the 1% level. But -5.51 is less than 5% (-5.09) and 10% (-4.83) critical values, so we reject H0 at the 5% and 10% levels. From this outcome, we conclude that the series is stationary at the 5% and 10% significance levels but non-stationary at the 1% level despite having a structural break in the trend component.

In Example 3 (Trend Only), the Zivot-Andrews (ZA) test statistic is -5.76. This statistic is compared to the critical values at various levels of significance: -5.58 (1%), -5.09 (5%), and -4.83 (10%). Since -5.76 is less than the 1% critical value (-5.58) and 5% (-5.09) critical values hence, we can reject the null at 1% and 5% levels. From this outcome, we conclude that the series is stationary at the 5% significance level indicating that the time series data exhibits stationarity with a structural break in both intercept and trend component.

The findings suggest that structural breaks in themselves are not adequate to prove stationarity in Examples 1 and 2, but permitting a break in intercept and trend (Example 3) is conclusive evidence of stationarity at the 5% level.

PANEL UNIT ROOT

IM-PESARAN-SHIN (IPS)

The Im-Pesaran-Shin (IPS) test is used to determine the presence of unit roots in the context of panel data. It accommodates individual unit root processes in individual cross-sectional units and tests for overall stationarity of the panel. Through this test, individual unit root processes within the cross-sectional units can be accepted or rejected when overall stationarity within the panel is tested.

Hypotheses of IPS Test

Null Hypothesis (H0): Unit root exists within the panel data series (the series is not stationary).
Alternative Hypothesis (H1): At least a few cross-sections in the panel are fixed.

Decision Rule

If the test statistic is smaller than the critical value, we reject H0, suggesting stationarity.

If the test statistic is larger than the critical value, we fail to reject H0, suggesting non-stationarity.

Table 5: IM-PESARAN-SHIN (IPS) test results

	Test Stats	1% CV	5% CV	10% CV
1. Level, No Trend	-1.76	-2.34	-1.99	-1.86
2. Level, with Trend	-2.66	-2.9	-2.63	-2.49
3. First difference, No Trend	-3.21	-2.34	-1.99	-1.86
4. First difference, With Trend	-3.76	-2.9	-2.63	-2.49

Example 1 (Level, No Trend): The Im-Pesaran-Shin (IPS) test statistic of -1.76 is higher than the 1% critical value of -2.34, 5% critical value of -1.99, and 10% critical value of -1.86. Because the test statistic is not higher than any of the critical values required for rejection, we do not reject the null hypothesis (H0), meaning that the panel data is non-stationary at level without a trend.

Example 2 (Level, With Trend): The Im-Pesaran-Shin (IPS) test statistic of -2.66 is higher than the 1% critical value of -2.9 (Fail to reject null), but less than 5% critical value of -2.63, 10% critical value of -2.49 (failed to reject null). Since the test stats -2.66 is lesser than the 5% and 10% critical values, we reject H0 at 5% level, meaning that the panel data is stationary at level with trend component.

Example 3 (First differenced, No Trend): The Im-Pesaran-Shin (IPS) test statistic of -3.21 is lower than the 1% critical value of -2.34, 5% critical value of -1.99, and 10% critical value of -1.86. This implies that the panel becomes stationary after being first differenced, suggesting that the original series is an integrated process.

Example 4 (First Difference, With Trend): The test statistic (-3.75) is smaller than the 1% (-2.9), 5% (-2.63), and 10% (-2.49) critical values, and thus there is a rejection of H0 at the 1% level. This indicates that with first differences and a trend, the panel data is stationary.

The findings indicate that the panel data is non-stationary at level (with and without trend). However, after being first differenced, the panel becomes stationary at the 1% significance level, verifying the existence of unit roots in level form and advocating for first-differencing in order to become stationary.

LEVIN-LIN-CHU (LLC)

Levin-Lin-Chu (LLC) Unit Root Test Decision Rule for LLC Test:

Null Hypothesis (H0): The panel data has a unit root (non-stationary).

Alternative Hypothesis (H1): The panel data is stationary.

Decision Rule

If the test statistic is smaller than the critical value, we reject H0 and conclude stationarity. If the test statistic is larger than the critical value, we fail to reject H0 and conclude non-stationarity.

Model	Test Stats	1% CV	5% CV	10% CV
Example 1 (Level, No Trend)	-3.11	-3.46	-2.88	-2.66
Example 2 (Level, With Trend)	-2.56	-3.46	-2.88	-2.66
Example 3 (First Difference, No Trend)	-4.00	-3.46	-2.88	-2.66
Example 4 (First Difference, With Trend)	-3.81	-3.46	-2.88	-2.66

Example 1 (Level, No Trend): The test statistic (-3.10) is larger than the 1% critical value (-3.45) but smaller than the 5% (-2.89) and 10% (-2.65) critical values. This implies that we reject H0 at the 5% level but not at the 1% level. There is moderate evidence that the series is stationary, but it is not highly significant, consider this result has non-stationary only.

Example 2 (Level, With Trend): The test statistic (-2.56) is larger than the 1% critical value (-3.45), 5% (-2.89) and 10% (-2.65) critical values. This implies that we failed to reject H0 at the 1%, 5% at 10% level. There is clear evidence that the series is non-stationary when trend component is included.

Example 3 (First differenced, No Trend): The test statistic (-4.00) is lesser than the 1% critical value (-3.45), 5% (-2.89) and 10% (-2.65) critical values. This implies that we can reject H0 at the 1%, 5% at 10% level. There is clear evidence that the series is stationary after first differencing when trend component is not included.

Example 3 (First differenced, No Trend): The test statistic (-3.81) is lesser than the 1% critical value (-3.45), 5% (-2.89) and 10% (-2.65) critical values. This implies that we can reject H0 at the 1%, 5% at 10% level. There is clear evidence that the series is stationary after first differencing when trend component is included.

Conclusion: At level form (without trend), there is significant evidence of stationarity (at 5%) but not at 1%. At level form (with trend), the panel is still non-stationary and needs transformation. After first differencing (with and without trend), the data is strongly stationary (at 1%), reaffirming that the panel exhibits an integrated process of order one (I (1)). For additional econometric analysis, first-differenced data must be applied in order to achieve stationarity and prevent spurious regression outcomes.

VI. DISCUSSION AND CONCLUSION

Stationarity is a basic requirement in time series modelling, especially in Vector Autoregression (VAR) and cointegration analysis, to ensure that the statistical properties of mean and variance are constant over time (Enders, 2008). Stationarity in a VAR model is crucial because non-stationary time series could lead to spurious regressions, and coefficient estimates become unstable (Sims, 1980). When variables are non-stationary but cointegrated, an error correction model (ECM) replaces VAR as it allows for both short-run dynamics and long-run equilibrium relations (Engle & Granger, 1987). In model forecasts like Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH), stationarity is critical. ARIMA needs differencing to attain stationarity prior to estimation (Box et al., 1976; Jenkins & Box, 1976; Box et al., 2015). Likewise, GARCH models, employed to model financial volatility, posit stationarity in variance (Bollerslev, 1986). Lacking stationarity, the validity of future predictions and confidence intervals in these models deteriorates (Tsay, 2010).

Furthermore, stationarity is important in macroeconomic and financial modelling, given that most economic time series, for instance, GDP, inflation, and stock prices, are unit-rooted (Nelson & Plosser, 1982). Stationarity detection helps determine whether shocks to an economic variable are permanent or temporary (Stock & Watson, 1999). In finance, stationarity in returns of assets is crucial for risk modelling, portfolio optimization, and market efficiency tests (Fama, 1970). The presence of non-stationarity in financial data often requires the application of cointegration methods or long-memory models for correct specification of persistent dependence (Granger, 1981). So, stationarity is a foundation for empirical work, on which rests the validity of econometric inference, stability of predictive models, and precision in macroeconomic and financial decision-making.

Time series analysis has been subject to numerous challenges, including in small samples, where sample size distortions and power weaknesses in unit root tests have emerged (Kapetanios et al., 2006; Otero & Smith, 2017). A lot of standard unit root tests, such as the Augmented Dickey-Fuller (ADF) test and Phillips-Perron (PP) test, have the propensity of exhibiting size distortions—rejecting the null hypothesis of a unit root inappropriately with very small samples (Harris, 1992). On the other hand, these tests are also low in power, not rejecting stationarity when the alternative holds (DeJong et al., 1992). To avoid these issues, researchers have used panel unit root tests like Im-Pesaran-Shin (IPS) and Levin-Lin-Chu (LLC), which utilize cross-sectional information to increase test power (Pesaran, 2007). Another major problem in time series modelling is the effect of structural breaks and nonlinearities (Perron, 1989). Most macroeconomic and financial time series experience shocks or regime shifts e.g., policy changes, financial crises, or pandemics that change their statistical properties. Most conventional unit root tests cannot tell the difference between a genuine unit root process and a stationary process with breaks (Zivot & Andrews, 1992). Structural break tests, such as the Bai-Perron (2003) test for multiple breaks and the Kapetanios, Shin, and Snell (KSS) test for nonlinear stationarity, address these limitations (Bai & Perron, 2003).

A long-standing debate exists between difference-stationary (DS) and trend-stationary (TS) processes (Nelson & Plosser, 1982). In a difference-stationary process, shocks have permanent effects, requiring differencing to achieve stationarity. Conversely, a trend-stationary process returns to a deterministic trend over time in that shocks only have transitory impacts (Perron, 1988). The difference has significant implications for macroeconomic modelling since the incorrect labelling of a series as DS rather than TS (or vice versa) can result in policy inaccuracies (Campbell & Mankiw, 1987). Empirical findings indicate that most economic variables, e.g., GDP and share prices, have mixed properties and hence need hybrid techniques like fractional integration models (Diebold & Rudebusch, 1991). To summarize, it is important to correct size distortions, structural breaks, and stationarity misclassifications in order to conduct solid time series analysis. Improved econometric methods, e.g., panel unit root tests, break-adjusted tests, and nonlinear models, offer more accurate understanding of economic and financial processes.

Developments in time series econometrics require continuing research on nonlinear unit root tests and machine learning-based methods to enhance the accuracy of forecasts and model robustness. Traditional unit root tests, such as the Augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) tests, assume linear stationarity; yet, most economic and financial real-world time series possess nonlinear behaviours due to asymmetric shocks, regime shifts, and threshold effects (Kapetanios et al., 2003). Researchers have subsequently developed nonlinear unit root tests, such as the Kapetanios, Shin, and Snell (KSS) test for ESTAR processes (Kapetanios et al., 2003) and the Enders-Granger test for TAR models (Enders & Granger, 1998). Theoretical work in the future can introduce hybrid models with structural breaks, nonlinearities, and time-varying parameters for increased robustness (Shin et al., 2014).

With the increasing provision of big data and computing capabilities, machine learning (ML) methods offer prospective substitutes for the identification of stationarity and the prediction of economic time series (Gu, Kelly, & Xiu, 2020). Methods like long short-term memory (LSTM) networks, recurrent neural networks (RNNs), and ensemble learning have the ability to identify intricate patterns and nonlinear relationships that are missed by conventional models (Makridakis et al., 2018). Nonetheless, ML-based methods need to be thoroughly validated since they have been criticized for being theoretically uninterpretable and inferentially weak compared to classical econometric models (Varian, 2014). Future studies need to involve the integration of econometric theory with ML algorithms, the construction of techniques for interpreting deep learning models, and the improvement of hybrid methods combining ML with standard time series methods (Medeiros et al., 2021).

Aside from methodological innovation, stationarity testing holds important policy ramifications for economic and financial analysis. Numerous macroeconomic policies are contingent on the assumptions of stationarity of indicators such as GDP growth, inflation, and interest rates. Policymakers have to consider permanent shocks instead of the assumption of mean reversion when a series is difference-stationary (DS) as opposed to trend-stationary (TS) (Nelson & Plosser, 1982). Misidentification of stationarity may result in improper monetary and fiscal policies, impacting inflation targeting, public debt management, and long-term economic planning (Perron, 1989). In financial markets, proper detection of stationarity is important for risk management, portfolio optimization, and derivative pricing models (Christensen & Prabhala, 1998). In summary, future research has to concentrate on the development of nonlinear unit root tests, the incorporation of machine learning techniques into time series analysis, and the improvement of policy models to guarantee sound economic forecasting and decision-making. Overcoming these challenges will reinforce the reliability of econometric models under structural breaks, nonlinearity, and high-dimensional data settings.

References

Bai, J., & Perron, P. (2003). Computation and analysis of multiple structural change models. *Journal of applied econometrics*, *18*(1), 1-22.

Bai, J., & Perron, P. (2006). Multiple structural change models: a simulation analysis. *Econometric theory and practice: Frontiers of analysis and applied research*, *1*, 212-237.

Baltagi, B. H., & Baltagi, B. H. (2008). *Econometric analysis of panel data* (Vol. 4, pp. 135-145). Chichester: John Wiley & sons.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, *31*(3), 307-327.

Box George, E. P., Jenkins Gwilym, M., Reinsel Gregory, C., & Ljung Greta, M. (1976). Time series analysis: forecasting and control. *San Francisco: Holden Bay*.

Jenkins, G. M., & Box, G. E. (1976). Time series analysis: forecasting and control. *(No Title)*.

Box, G. E., Jenkins, G. M., Reinsel, G. C., & Ljung, G. M. (2015). *Time series analysis: forecasting and control*. John Wiley & Sons.

Breitung, J. (2001). The local power of some unit root tests for panel data. In *Nonstationary panels, panel cointegration, and dynamic panels* (pp. 161-177). Emerald Group Publishing Limited.

Breitung, J., & Pesaran, M. H. (2008). Unit roots and cointegration in panels. In *The econometrics of panel data: Fundamentals and recent developments in theory and practice* (pp. 279-322). Berlin, Heidelberg: Springer Berlin Heidelberg.

Breitung, J., & Pesaran, M. H. (2005). Unit roots and cointegration in panels.

Campbell, J. Y., & Mankiw, N. G. (1987). Are output fluctuations transitory? *The Quarterly Journal of Economics*, *102*(4), 857-880.

Glynn, J., Perera, N., & Verma, R. (2007). Unit root tests and structural breaks: A survey with applications. *Revista de Métodos Cuantitativos para la Economía y la Empresa*, *3*, 63-79.

Stock, J. H. (1994). Unit roots, structural breaks and trends. *Handbook of econometrics*, *4*, 2739-2841.

Byrne, J. P., & Perman, R. (2006). Unit roots and structural breaks: a survey of the literature.

Choi, I. (2001). Unit root tests for panel data. *Journal of international money and Finance*, *20*(2), 249-272.

Christensen, B. J., & Prabhala, N. R. (1998). The relation between implied and realized volatility. *Journal of financial economics*, *50*(2), 125-150.

DeJong, D. N., Nankervis, J. C., Savin, N. E., & Whiteman, C. H. (1992). The power problems of unit root test in time series with autoregressive errors. *Journal of econometrics*, *53*(1-3), 323-343.

Dickey, D. A., & Fuller, W. A. (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*, *74*(366a), 427-431.

Dickey, D. A., & Fuller, W. A. (1981). Likelihood ratio statistics for autoregressive time series with a unit root. *Econometrica: journal of the Econometric Society*, 1057-1072.

Diebold, F. X. (2015). Elements of Forecasting. Cengage Learning.

Francis X. Diebold & Glenn D. Rudebusch (1991). Is Consumption Too Smooth? Long Memory and The Deaton Paradox. The Review of Economics and Statistics, 106(4), 861-880.

Elliott, G., Rothenberg, T. J., & Stock, J. H. (1992). Efficient tests for an autoregressive unit root.

Enders, W. (2008). *Applied econometric time series*. John Wiley & Sons.

Enders, W., & Granger, C. W. J. (1998). Unit-root tests and asymmetric adjustment with an example using the term structure of interest rates. *Journal of Business & Economic Statistics*, *16*(3), 304-311.

Engle, R. F., & Granger, C. W. (1987). Co-integration and error correction: representation, estimation, and testing. *Econometrica: journal of the Econometric Society*, 251-276.

Fama, E. F. (1970). Efficient capital markets. *Journal of finance*, *25*(2), 383-417.

Fuller, W. A. (2009). *Introduction to statistical time series*. John Wiley & Sons.

Granger, C. W. (1981). Some properties of time series data and their use in econometric model specification. *Journal of econometrics*, *16*(1), 121-130.

Granger, C. W. J. (2001). Some properties of time series data and their use in econometric model specification. In *Essays in econometrics: collected papers of Clive WJ Granger* (pp. 119-128).

Granger, C. W., & Newbold, P. (1974). Spurious regressions in econometrics. *Journal of econometrics*, *2*(2), 111-120.

CWJ, G. (1974). Spurious regression in econometrics. *J Economet*, *48*, 283-307.

Gu, S., Kelly, B., & Xiu, D. (2020). Empirical asset pricing via machine learning. *The Review of Financial Studies*, *33*(5), 2223-2273.

Gujarati, D. N., & Porter, D. C. (2009). Basic econometrics (5th ed.). McGraw-Hill.

Gujarati, D. N. (2002). Basic Econometrics 4th ed. McGraw-Hill. Add this as well

Hamilton, J. D. (2020). *Time series analysis*. Princeton university press.

Harris, R. I. (1992). Testing for unit roots using the augmented Dickey-Fuller test: Some issues relating to the size, power and the lag structure of the test. *Economics letters*, *38*(4), 381-386.

Harris, R., & Sollis, R. (2003). *Applied Time Series Modelling and Forecasting*. Wiley

Hyndman, R. J., & Athanasopoulos, G. (2018). *Forecasting: principles and practice*. OTexts.

Im, K. S., Pesaran, M. H., & Shin, Y. (2003). Testing for unit roots in heterogeneous panels. *Journal of econometrics*, *115*(1), 53-74.

Johansen, S. (1991). Estimation and hypothesis testing of cointegration vectors in Gaussian vector autoregressive models. *Econometrica: journal of the Econometric Society*, 1551-1580.

Kapetanios, G., Shin, Y., & Snell, A. (2003). Testing for a unit root in the nonlinear STAR framework. *Journal of econometrics*, *112*(2), 359-379.

Kwiatkowski, D., Phillips, P. C., Schmidt, P., & Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of econometrics*, *54*(1-3), 159-178.

Levin, A., Lin, C. F., & Chu, C. S. J. (2002). Unit root tests in panel data: asymptotic and finite-sample properties. *Journal of econometrics*, *108*(1), 1-24.

Lumsdaine, R. L., & Papell, D. H. (1997). Multiple trend breaks and the unit-root hypothesis. *Review of economics and Statistics*, *79*(2), 212-218.

MacKinnon, J. G. (1996). Numerical distribution functions for unit root and cointegration tests. *Journal of applied econometrics*, *11*(6), 601-618.

Maddala, G. S., & Wu, S. (1999). A comparative study of unit root tests with panel data and a new simple test. *Oxford Bulletin of Economics and statistics*, *61*(S1), 631-652.

Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2018). Statistical and Machine Learning forecasting methods: Concerns and ways forward. *PloS one*, *13*(3), e0194889.

Medeiros, M. C., Vasconcelos, G. F., Veiga, Á., & Zilberman, E. (2021). Forecasting inflation in a data-rich environment: the benefits of machine learning methods. *Journal of Business & Economic Statistics*, *39*(1), 98-119.

Narayan, P. K., & Popp, S. (2010). A new unit root test with two structural breaks in level and slope at unknown time. *Journal of Applied Statistics*, *37*(9), 1425-1438.

Nelson, C. R., & Plosser, C. R. (1982). Trends and random walks in macroeconmic time series: some evidence and implications. *Journal of monetary economics*, *10*(2), 139-162.

Ng, S., & Perron, P. (2001). Lag length selection and the construction of unit root tests with good size and power. *Econometrica*, *69*(6), 1519-1554.

Perron, P. (1988). Trends and random walks in macroeconomic time series: Further evidence from a new approach. *Journal of economic dynamics and control*, *12*(2-3), 297-332.

Perron, P. (1986). *Trends and random walks in macroeconomic time series: Further evidence from a new approach* (No. 8650). Add this as well

Perron, P. (1997). Further evidence on breaking trend functions in macroeconomic variables. *Journal of econometrics*, *80*(2), 355-385.

Pesaran, M. H. (2007). A simple panel unit root test in the presence of cross-section dependence. *Journal of applied econometrics*, *22*(2), 265-312.

Phillips, P. C. (1987). Time series regression with a unit root. *Econometrica: Journal of the Econometric Society*, 277-301.

Phillips, P. C., & Perron, P. (1988). Testing for a unit root in time series regression. *biometrika*, *75*(2), 335-346.

Phillips, P. (1988). Testing for unit roots in time series regression. *Biometrika*, *71*, 599-607. Add this as well

Priestley, M. B. (1981). Spectral Analysis and Time Series. Academic Press.

Schwert, G. W. (2002). Tests for unit roots: A Monte Carlo investigation. *Journal of business & economic statistics*, *20*(1), 5-17.

Shiller, R. J., & Perron, P. (1985). Testing the random walk hypothesis: Power versus frequency of observation. Economics Letters, 18(4), 381-386.

Kapetanios, G., Shin, Y., & Snell, A. (2006). Testing for cointegration in nonlinear smooth transition error correction models. *Econometric Theory*, *22*(2), 279-303.

Otero, J., & Smith, J. (2017). Response surface models for OLS and GLS detrending-based unit-root tests in nonlinear ESTAR models. *The Stata Journal*, *17*(3), 704-722.

Shin, Y., Yu, B., & Greenwood-Nimmo, M. (2014). Modelling asymmetric cointegration and dynamic multipliers in a nonlinear ARDL framework. *Festschrift in honor of Peter Schmidt: Econometric methods and applications*, 281-314.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica: journal of the Econometric Society*, 1-48.

Stock, J. H., & Watson, M. W. (1988). Testing for common trends. *Journal of the American statistical Association*, *83*(404), 1097-1107.

Stock, J. H., & Watson, M. W. (1999). Forecasting inflation. *Journal of monetary economics*, *44*(2), 293-335.

Stock, J. H., & Watson, M. W. (2007). Introduction to Econometrics, ed. *Pearson*.

Stock, J. H., & Watson, M. W. (2011). Introduction to econometrics, Addison-Wesley Series in Economics.

Stock, J. H., & Watson, M. W. (2015). Introduction to econometrics (3rd updated edition). *Age (X3)*, *3*(0.22).

Sul, D., Phillips, P. C., & Choi, C. Y. (2005). Prewhitening bias in HAC estimation. *Oxford Bulletin of Economics and Statistics*, *67*(4), 517-546.

Tsay, R. S. (2005). *Analysis of financial time series*. John wiley & sons.

Varian, H. R. (2014). Big data: New tricks for econometrics. *Journal of economic perspectives*, *28*(2), 3-28.

Zhang, G., Zhang, J., & Zhang, J. (2020). "Deep Learning for Time Series Forecasting in Financial Markets." IEEE Transactions on Neural Networks and Learning Systems, 31(10), 3791-3803.

Zivot, E., & Andrews, D. W. K. (1992). "Further evidence on the Great Crash, the oil-price shock, and the unit-root hypothesis." Journal of Business & Economic Statistics, 10(3), 251-270. Replace with the following

Zivot, E., & Andrews, D. W. K. (2002). Further evidence on the great crash, the oil-price shock, and the unit-root hypothesis. *Journal of business & economic statistics*, *20*(1), 25-44.