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Determining the Pension Benefit Obligation of a Defined Benefit Plan: Applying a Multivariate ARIMA Stochastic Model

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ABSTRACT

In this study, we examine the robustness of fit for a multivariate and an autoregressive integrated moving average model to a data sample time series type. The sample is a recurrent actuarial data set for a 10-year horizon. We utilize this methodology to contrast with stochastic models to make projections beyond the data horizon. Our key results suggest that both types of models are useful for making predictions of actuarial liability levels given by PBO Projected Benefit Obligations on and off the horizon of the sample time series. As we have seen in prior research, the use of multivariate models for control and auditing purposes is widely recommended. Fast and reliable statistical estimates are desirable in all cases, whether for audit purposes or to verify and validate miscellaneous actuarial results.

Keywords: Multivariate Regression, ARIMA Model, Regressive Model, Pensions.

1. Introduction and Objective

In this study, we analyze the goodness of fit using a multivariate regression model and a self-regressive autoregressive integrated moving average (ARIMA)-type regression model to our data sample, which is basically recurrent valuations from the analytical point of view for a 10-year horizon. Those results are then contrasted with the results of a regression model for series data within the sample ultimately, and another ARIMA to make projections beyond the data horizon. By using lagged moving averages to smooth time series data, autoregressive integrated moving average (ARIMA) models are used to predict future values based on past values. In other words, autoregressive models implicitly assume that the future will resemble the past. In theory, ARIMA models are the most general class of models for forecasting a time series which can be made to be "stationary" via such tools as differencing, or nonlinear transformations such as logging or deflating. If a time series random variable has statistical properties that are all constant over time, it is stationary. Characteristics of a stationary series include having no trend, short-term random time patterns always looking the same in a statistical sense, and variations around its mean having a constant amplitude. An ARIMA model can be described as a filter used to separate the signal from the noise.

In a 2018 study by Diz and Query, a multivariate regression model is determined that allows computation of the actuarial liability of social benefits using a group of potential predictors. In general, the previous or independent predictors are the same as those utilized in an actuarial valuation of labor commitments. Several linear and non-linear models are considered and tested in this study. Among the most important findings of this research is that the PBO or Actuarial Liability depends fundamentally on a linear basis of two fundamental variables in the quantification of Social Benefits -the Guarantees and the Social Benefits to Pay (PSP).

2. Motivation for Study

In actuarial practice, the need to make estimates of the level of liabilities within and outside a time horizon is very common given a series of sample observations derived from the realization of recurrent actuarial assessments. In this paper, two stochastic models are developed; to make predictions within the horizon of the sample time series and projections outside it. Predictions within the sample domain use a multivariate regression model based on one or more predictor variables of the best possible fit. Outside the domain, an ARIMA-type self-regulating stochastic model was adjusted to model liabilities levels for five future years.

3. Methodology Use for Determining Actuarial Liability

Description of actuarial model variables:

3.1 Warranty: Amount of Social Benefits type defined benefit plans applicable in Venezuelan labor law.

$$G = \sum S_t z_t$$

S_t: Salary at time t
z_t: Profit rate applicable at time t

3.2 Retroactive Benefits: This benefit is exponential in nature, and the amount is calculated by collecting all the services accumulated to date by the respective salary.

$$R_t = S_t t$$

S_t: Salary
t: years of accumulated service

3.3 Differential Benefits: Represents the difference in Benefits between retroactive accounting and warranty.

$$D_{t=}R_t - G_t$$

D_ (t =) Earnings spread

3.4 Service: Seniority in the company

$$x - y$$

$x: Actual\ Age$

$y: Entry\ Age$

3.5 Lottt Service: Recognizable or creditable service for retroactivity purposes.

$$Serv - \Delta$$
$$\Delta: Restricciones$$

LOTTT refers to the Organic labor law in Venezuela. LOTTT SERVICES means accredited service (Seniority) under LOTTT for retroactivity calculation.

3.6 Differential Benefit: The maximum positive difference between retroactivity and warranty.

$$B_t = MAX(R_t - G_t, 0)$$

3.7 Actuarial Liabilities (Unit Projected Benefit):

$PBO_t = B_t A_t$

A_t : Expected present value of the unit benefit payment
$A_t = f(i, \Delta, tp_x q_x)$
i: Interest rate
s: Rate of salary increase
$tp_x q_x$: Probability of Unit Payment

4. Demographic and Salary Descriptions of the Sample

4.1 The zero-year sample is described below.

SAMPLE	30/06/2020
POPULATION	134.00
CURRENT AGE (AVERAGE)	41.69
CURRENT SERVICE (AVERAGE)	11.32
INTEGRAL SALARY (AVERAGE)	68.515.703.36
PAYROLL/MONTH	9.181.104.250.64
WARRANTY PS	5.259.029.228.22
RETROACTIVE PS	107.523.454.040.37
PS TO PAY	107.523.454.040.37
DIF TO PAY VS GUARANTEES	102.264.424.812.15

4.2 Actuarial valuations were made under the Projected Benefit Credit Unit Method. In general, actuarial obligations and annual service costs are modeled (PBO and CS) over the age of 10.

	PBO	CS
YEAR 0	599.581.716.879.925	158.390.553.792.612
YEAR 1	218.504.785.809.353	372.689.113.444.659
YEAR 3	7.164.245.675.695	914.233.917.423.892
YEAR 3	193.410.357.839.552	183.989.742.883.175
YEAR 4	415.185.126.949.638	367.284.435.386.488
YEAR 5	798.854.276.562.709	594.322.070.312.557
YEAR 6	127.672.317.372.548	723.696.004.939.019
YEAR 7	176264597955.79	777.578.845.397.996
YEAR 8	214.968.888.852.038	814.681.692.195.149
YEAR 9	249.996.814.127.197	810.811.800.383.123
YEAR 10	284980458379.68	794.778.715.261.004

From the valuation results of the initial year, the 10 successive actuarial valuations are developed and the following descriptive statistics of future years are found.

	TOTALS			
	PBO			
	TOTAL	MIN	MAX	DESV. TIP
YEAR 0	80343950061.91	26847120.56	7927232001.51	756.769.421.733.148
YEAR 1	292.796.412.984.533	318.216.369.781.985	273.839.681.788.948	257.550.745.029.459
YEAR 3	960008920543.13	151.923.449.559.289	874.143.417.932.705	81.170.088.298.588
YEAR 3	2.591.698.795.050	480.944.269.931.031	231.628.064.673.957	212.777.804.880.476
YEAR 4	5563480701125.15	111.587.464.495.244	489.993.791.402.217	445.988.391.154.724
YEAR 5	10704647305940.3	227.906.257.701.688	931.522.560.022.985	841.278.635.020.668
YEAR 6	17108090527921.4	381.700.496.344.534	1473919782921.89	132.197.346.899.611
YEAR 7	23619456126075.8	547.320.011.543.991	2019035244005.26	179.965.244.993.454
YEAR 8	28805831106173.1	687.620.480.392.791	2444409060434.9	216.824.770.843.672
YEAR 9	33499573093044.4	820.001.298.440.968	2828611902071.11	249.655.073.896.059
YEAR 10	38187381422877.1	954.091.067.078.157	3211797561920.43	282.227.235.423.713

5. Formulation of Statistical Models

Two models are adjusted from the results, and are described in this section:

5.1 Multivariate Regression Model for predictions within the domain (10 years).

There is Little extant research about this particular subject and this is one of the reasons for our investigation. Most of the principal auditing companies worldwide engage statisticians using multivariate regression analysis in order to check the actuarial valuations and results. The methodology is less expensive, quicker, and in terms of order of magnitude, the figures are basically the same. These statistical models allow us to contrast results based on evidence.

<u>MULTIPLE REGRESSION - PBO</u>

Dependent variable: pbo
Independent variables:
  service (serv)
  tint (229%)

NOTE: Tint means rate of interest; in Spanish it is tasa de interés, in short, “tint”

5.2 Multiple Regression Statistics

		Standard	*Statistical*	
Parameter	***Estimate***	***Error***	***T***	***P-value***
constant	-1.15976E14	1.34678E13	-8.61138	0.0001
service	7.3446E12	7.02078E11	10.4612	0.0000
tint	1.31754E13	2.65369E12	4.96494	0.0016

5.3 Variance Analysis

fountain	***Sum of Squares***	***Gl***	***Medium Square***	***F-Reason***	***P-value***
model	1.41345E27	2	7.06724E26	206.78	**0.0000**
residue	2.39243E25	7	3.41776E24		
Total (Corr.)	1.43737E27	9			

R-square-98.3356 percent
R-square (adjusted for g.l.) - 97.86 percent
Standard est error. 1 .84872E12
Average Absolute Error - 1,34777E12
Durbin-Watson Statistician s 1,38844 (Ps 0,0190)
Delayed waste autocorrelation 1 x 0.111525

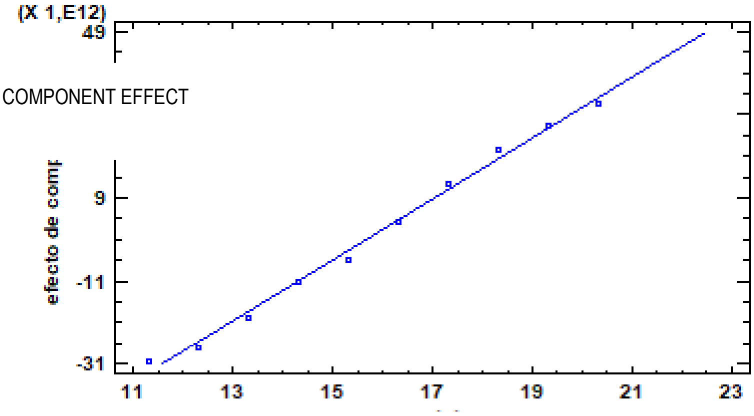
The output shows the results of adjusting a multiple linear regression model to describe the relationship between pbo and 2 independent variables. The equation of the fitted model is:

$$pbo = 1.15976E14 + 7.3446E12*service + 1.31754E13*tint$$

Since the P-value in the ANOVA table is less than 0.05, there is a statistically significant relationship between variables with a 95.0% confidence level. The R-Square statistic indicates that the model thus adjusted explains 98.34% of the variability in pbo. The adjusted R-Square statistic, which is more appropriate for comparing models with a different number of independent variables, is 97.86%. The standard estimated error shows that the standard deviation of the residuals is 1.84872E12. This value can be used to construct boundaries for new observations by selecting the Reports option from the text menu. The average absolute error (MAE) of 1.34777E12 is the average value of the residuals. The Durbin-Watson (DW) statistic examines the residuals to determine if there are any significant correlations based on the order in which they are presented in the data file. Since the P-value is less than 0.05, there is an indication of a possible serial correlation with a 95.0% confidence level. Plot the residuals versus the row number to see if there are any patterns that can be detected.

To determine whether the model can be simplified, notice that the highest P-value of the independent variables is 0.0016, which corresponds to tint. Since the P-value is less than 0.05, that term is statistically significant with a confidence level of 95.0%. Consequently, you probably wouldn't want to remove any variables from the model.

PBO RESIDUE COMPONENT GRAPH



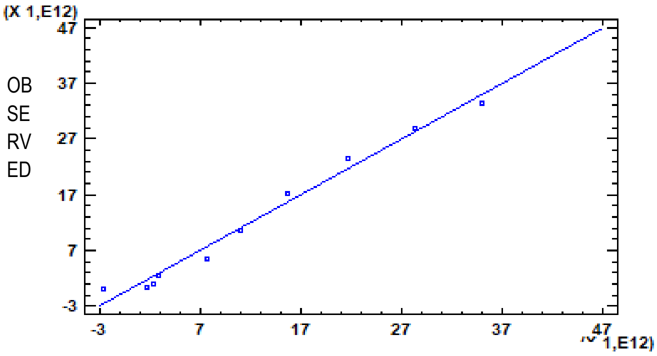
SERVICE

Additional ANOVA for Variables in Adjusted Order

Fountain	*Sum of squares*	*Gl*	*Mean Square*	*F-Ratio*	*P-Value*
Years of service	1.3292E27	1	1.3292E27	388.91	0.0000
Tint	8.42497E25	1	8.42497E25	24.65	0.0016
Model	1.41345E27	2			

This table shows the statistical significance of each variable as it was added to the model. You can use this table to help you determine if the model can be simplified, especially if you are fitting a polynomial.

PBO CHART



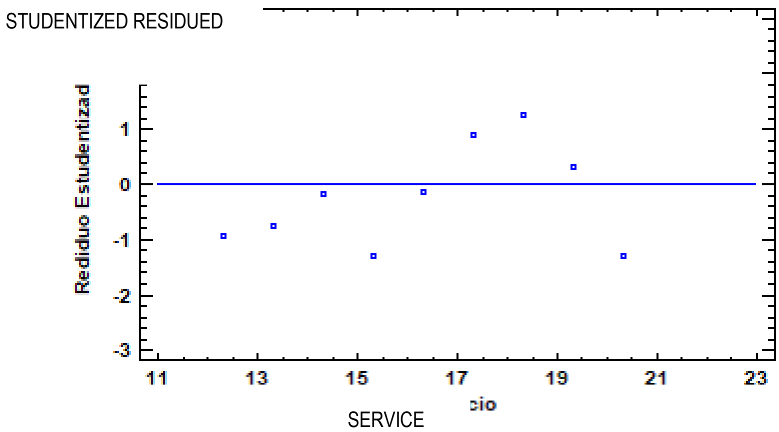
PREDICTED

95.0% CONFIDENCE INTERVALS FOR COEFFICIENT ESTIMATES

		Standard		
Parameter	*Estimate*	error	*Lower limit*	*Upper limit*
Constant	-1.15976E14	1.34678E13	-1.47822E14	-8.41297E13
Yearsofservice	7.3446E12	7.02078E11	5.68445E12	9.00476E12
tint	1.31754E13	2.65369E12	6.9004E12	1.94504E13

This table shows 95.0% confidence intervals for the coefficients in the model. Confidence intervals show how accurately the coefficients can be estimated given the amount of data available, and the level of noise that is present.

RESIDUAL CHART

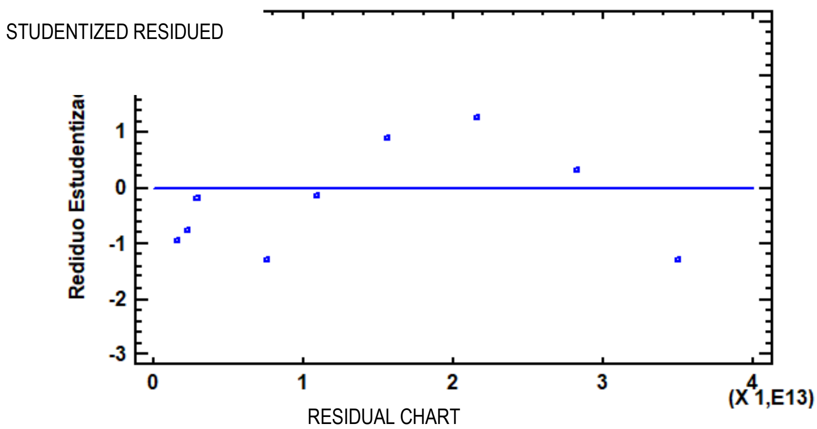


Correlation matrix for coefficient estimates

	Constant	Service	tint
Constant	1.0000	-0.9977	-0.9701
Years of service	-0.9977	1.0000	0.9572
tint	-0.9701	0.9572	1.0000

This table shows the estimated correlations between the coefficients in the fitted model. These correlations can be used to detect the presence of severe multicollinearity, that is, a correlation between the predictor variables. In this case, there is 1 correlation with an absolute value greater than 0.5 (not including the constant term).

RESIDUAL CHART



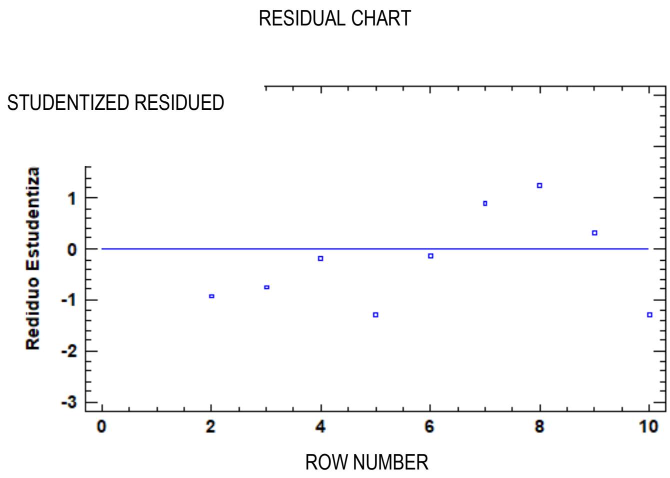
PBO Regression Results

	Tight	*Error Est.*	*Bottom 95.0%*	*Top 95.0%*	*Bottom 95.0%*
Row		*LC for Forecast*	*LC for Forecast*	*LC for Forecast*	*LC for the Average*

	Top 95.0%
row	*LC for the Average*

This table contains information about pbo which was generated using the fitted model. The table includes:

(1) the predicted pbo values using the fitted model
(2) the standard error to reach the predicted value
(3) 95.0% forecast intervals for new observations
(4) 95.0% confidence intervals for the mean response



Atypical Data (Waste)

				Residual
Row	*Y*	*Predicted Y*	*Residual*	*Studentized*
1	8.0344E10	-2.60278E12	2.68312E12	2.76

The table of outliers lists all observations that have Studentized residues greater than 2, in absolute value. The Studentized residuals measure how many standard deviations each observed pbo value is defined from the fitted model, using all data except that observation. In this case, there is a Studentized residue greater than 2, but none greater than the influential points table lists all observations that have influence values greater than 3 times that of an average point in the data, or that have an unusual value of DFITS. Influence Value is a statistic that measures how influential each observation is in determining the coefficients of the estimated model. DFITS is a statistic that measures how much-estimated coefficients could change if observation were removed from the dataset. In this case, an average point in the data would have an influence value equal to 0.3. There are no points with more than 3 times the average influence value. There are 2 data with unusually large DFITS values.

5.4 Automatic forecasting model – PBO

AUTOMATIC FORECASTS - PBO

Data/Variable: pbo

Number of observations = 11
Initial Index = 1.0
Sample Interval = 1.0

Forecast Summary
Selected forecast model: ARIMA (0, 2, 2)
Number of forecasts generated: 5
Number of periods retained for validation: 0

Statistical	*Period of estimate*
RMSE	6.33272E11
dude	4.72184E11
ASM	9,02893
me	1.68334E11
Mpe	7,65961

ARIMA Model Summary

Parameter	*Estimated*	*Estimation Error*	*t*	*P-value*
MA(1)	-1.16199	0.27106	-4,28684	0,003625
MA(2)	-0,860467	0,168472	-5,10749	0,001388

Historical Forecast: yes
Estimated white noise variance = 4.03345E23 with 7 degrees of freedom
Estimated standard deviation of white noise = 6.35094E11

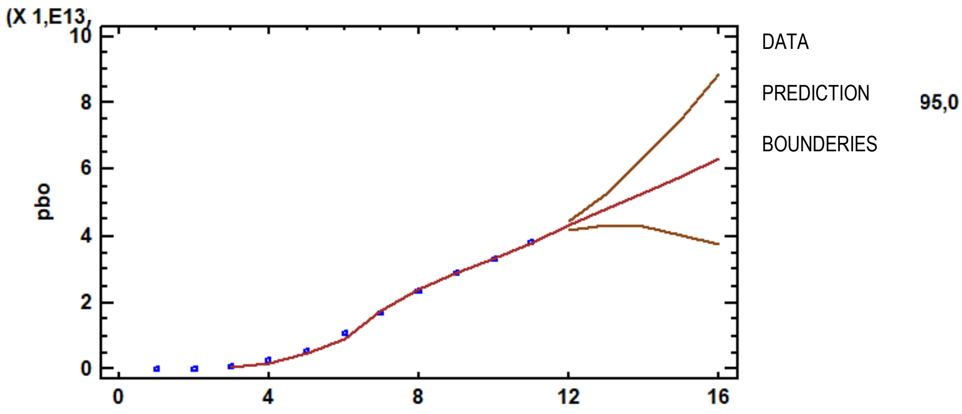
Number of iterations: 5
This procedure predicts future pbo values. The data covers 11 time periods. Currently, the model of an integrated autoregressive moving average (ARIMA) has been selected. This model assumes that the best forecast available for future data is given by the parametric model that relates the most recent value to the previous values and noise.

The output summarizes the statistical significance of the terms in the forecasting model. Terms with P-values less than 0.05 are statistically nonzero with a confidence level of 95.0%. The P-value for the term MA(2) is less than 0.05, so it is statistically different from 0. The estimated standard deviation of the input white noise is equal to 6.35094E11.

This table also summarizes the performance of the currently selected model in fitting historical data. It shows:
(1) the root of the mean square error (RMSE)
(2) the mean absolute error (MAE)
(3) the average absolute error rate (ASM)
(4) the mean error (ME)
(5) the average error rate (MPE)

Each of the statistics is based on the one-forward forecasting errors, which are the differences between the data at time t and the predicted value at time t-1. The first three statistics measure the magnitude of errors. A better model would give a smaller value. The last two statistics measure bias. A better model would give a value closer to 0.

PBO AS A FUNCTION OF TIME CHART



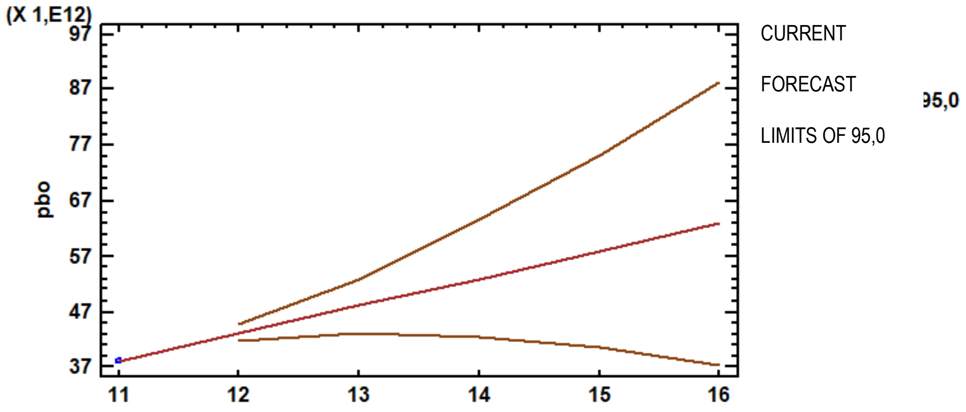
Forecast Table for pbo
Model: ARIMA (0,2,2)

Period	*Data*	*Forecast*	*Residual*
1.0	8.0344E10		
2.0	2.92796E11		
3.0	9.60009E11	6.39546E11	3.20463E11
4.0	2.5917E12	2.01939E12	5.72313E11
5.0	5.56348E12	5.16416E12	3.99321E11
6.0	1.07046E13	9.49173E12	1.21292E12
7.0	1.71081E13	1.75988E13	-4.9073E11
8.0	2.36195E13	2.3985E13	-3.6553E11
9.0	2.88058E13	2.92838E13	-4.7799E11
10.0	3.34996E13	3.31223E13	3.77314E11
11.0	3.81874E13	3.82205E13	-3.30757E10

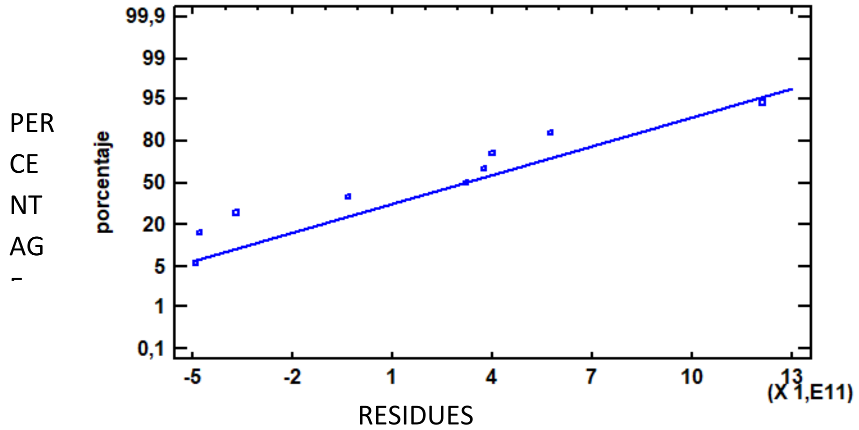
		Limit at 95.0%	*Limit at 95.0%*
Period	*Forecast*	*Lower*	*Upper*
12.0	4.31614E13	4.16597E13	4.46632E13
13.0	4.8107E13	4.31266E13	5.30874E13
14.0	5.30526E13	4.25139E13	6.35912E13
15.0	5.79982E13	4.06132E13	7.53832E13
16.0	6.29437E13	3.76547E13	8.82328E13

This table shows the predicted values for pbo. During the period where data are available, the predicted values of the fitted model and the residuals (data-forecast) are also displayed. For time periods beyond the time series, the 95.0% prediction limits for forecasts are displayed. These limits show where the true value of the data might be, at the selected future time, with 95.0% confidence, assuming that the fitted model is appropriate for the data.

FORECAST CHART FOR PBO



NORMAL PROBABILITY CHART FOR



ESTIMATED AUTOCORRELATIONS FOR ATYPICAL VALUES

Data variable: pbo

Model: ARIMA(0,2,2)

			Limit at 95.0%	*Limit at 95.0%*
Lag	*Autocorrelation*	*Error Estimation*	*Lower*	*Upper*
1	0,0890788	0,333333	-0,653323	0,653323
2	0,0751528	0,335968	-0,658486	0,658486
3	-0,366741	0,337831	-0,662137	0,662137

This table shows the estimated autocorrelations between the residuals at different delays. The delaying autocorrelation coefficient k measures the correlation between residuals at time t and time t-k. Probability limits of 95.0% around 0 are also shown. If the probability limits to a particular delay do not contain the estimated coefficient, there is a statistically significant correlation to that delay to the 95.0% confidence level. In this case, none of the 24 autocorrelation coefficients is statistically significant, implying that the time series may well be completely random (white noise).

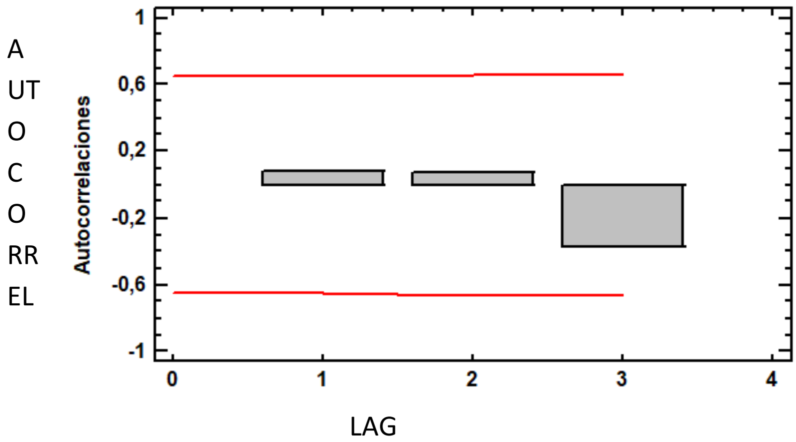
ESTIMATED SELF-CORRELATIONS FOR ATYPICAL VALUES

Data variable: pbo
Model: ARIMA (0,2,2)

			limit in 95,0%	limit in 95,0%
Lag	*Auto correlation*	*Error Estd.*	lower	higher
1	0.0890788	0.333333	-0.653323	0.653323
2	0.0751528	0.335968	-0.658486	0.658486
3	-0.366741	0.337831	-0.662137	0.662137

This table shows the estimated auto correlations between the residuals at different lags. The lagged auto correlation coefficient k measures the correlation between the residuals at time t and time t-k. 95.0% probability limits around 0 are also shown. If the probability limits at a particular delay do not contain the estimated coefficient, there is a statistically significant correlation to that delay at the 95.0% confidence level. In this case, none of the 24 auto correlation coefficients is statistically significant, implying that the time series may well be completely random (White noise).

RESIDUALS AUTOCORRELATION CHART



Conclusions

Based on the above analyses, it is apparent that both types of models are useful for making predictions of actuarial liability levels given by PBO Projected Benefit Obligations on and off the horizon of the sample time series. As we have seen in prior research, (Baker, 2003), (Godwin, et al. (1996) (Lin, 2000) and Wu, et al. (2013), the use of multivariate models for control and auditing purposes is widely recommended. Fast and reliable statistical estimates are desirable in all cases, whether for audit purposes or to verify and validate miscellaneous actuarial results.

The main contribution of the paper is that through both models reasonably good approximate results can be given, without the need to carry out recurrent actuarial assessments whose cost can be substantial, in the event that there is a need to make a significant number of them. In addition, the response time of regular actuarial valuations could be a little longer and consequently obtain extemporaneous results relatively close to those obtained in the valuations.

Within what might be called limitations, models would not consider an analysis of intervention of structural changes in benefits for obvious reasons. Obviously, new models could be adjusted in case this happens.

When negotiations on traded collective agreements are being discussed, this tool is of particular importance, because it is a process of strategic negotiation, timely and quality decision-making is invaluable.

Both the multivariate regression and autoregressive modeling yield very reasonable estimates. This would undoubtedly allow for impact forecasts with changes in the interest rate. For example, at the level of multivariate regression.

However, when we analyze the ARIMA models, the estimates that the model throws outside the time horizon show the potential risks in the confidence bands of the predictions at 95% statistical confidence. This is of the utmost importance in order to administer a benefit plan and define risk control actions.

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APPENDICES
Appendix I

Demographics

GARANTEE SOCIAL INDEMNITIES	TOTAL	80343950061.91
	PROM	599.581.716,88
	VAR	577.005.972.389.650.000
	MIN	26847120.56
	MAX	7927232001.51
	DESV. TIP.	756.769.421.733.148
CS	TOTAL	21224334208.21
	PROM	15.839.055.792.612
	VAR	4.539.122.834.672.520
	MIN	73032176.76
	MAX	838779267.18
	DESV. TIP.	671.211.500.472.653
WARRANTY	TOTAL	52.590.292.282.163
	PROM	392.464.867.777.336
	VAR	491.287.625.405.824
	MIN	165.961.205.236.849
	MAX	157.016.325.772.231
	DESV. TIP.	220.821.488.955.014
RETROACTIVE	TOTAL	107.523.454.040.368
	PROM	80.241.383.612.215
	VAR	878.356.628.178.672.000
	MIN	62.632.947
	MAX	963.536.370.079.461
	DESV. TIP.	933.703.233.350.074
DIFFERENTIAL	TOTAL	102.264.424.812.152
	PROM	763.167.349.344.417
	VAR	854.625.237.800.448.000
	MIN	308.168.047.871.755
	MAX	947.834.737.502.238
	DESV. TIP.	921.003.494.845.896
YEARS OF SERVICE	PROM	113.224.626.865.672
	VAR	680.892.593.031.085
	MIN	0.92
	MAX	40.07
	DESV. TIP.	822.077.432.498.539
CREDIT SERVICE	PROM	108.582.089.552.239
	VAR	517.466.614.296.936
	MIN	1
	MAX	23
	DESV. TIP.	71.666.234.946.833
SALARY	TOTAL	918.110.425.064.173
	PROM	685.157.033.629.979
	VAR	1.067.512.847.974.780
	MIN	335.357.512.421.092
	MAX	418.928.856.556.287
	DESV. TIP.	325.506.733.291.943
BENEFIT (X)	TOTAL	753.033.765.091.048
	PROM	561.965.496.336.603
	VAR	17.220.865.078.545.300
	MIN	602.973.606.091.617
	MAX	149.912.707.065.859
	DESV. TIP.	130.737.718.964.531

APPENDIX II

Actuarial Assumptions and Assumptions for 10 Years of Valuations

YEARS	AVERAGE	ROTATION	TOTAL ROTATION
23	0.0003245	0.1699416	0.1702661
24	0.000341	0.16122	0.161561
25	0.0003585	0.152946	0.1533045
26	0.000378	0.1450966	0.1454746
27	0.000398	0.13765	0.138048
28	0.0004215	0.1305856	0.1310071
29	0.000446	0.1238838	0.1243298
30	0.0004745	0.1175258	0.1180003
31	0.0005045	0.1114942	0.1119987
32	0.0005375	0.1057722	0.1063097
33	0.000574	0.1003438	0.1009178
34	0.000614	0.095194	0.095808
35	0.000668	0.0903084	0.0909764
36	0.0007045	0.0856738	0.0863783
37	0.0007505	0.0812768	0.0820273
38	0.000806	0.0771056	0.0779116
39	0.0008725	0.0731484	0.0740209
40	0.0009515	0.0693942	0.0703457
41	0.001043	0.0658328	0.0668758
42	0.001151	0.0624542	0.0636052
43	0.001278	0.059249	0.060527
44	0.0014255	0.0562082	0.0576337
45	0.0015965	0.0533236	0.0549201
46	0.001794	0.0505868	0.0523808
47	0.0020135	0.0479906	0.0500041
49	0.002509	0.0431912	0.0457002
50	0.002778	0.0409746	0.0437526
51	0.0030585	0.0388716	0.0419301
52	0.0033515	0.0368768	0.0402283
53	0.0036595	0.0349842	0.0386437
54	0.0039875	0.0331888	0.0371763
55	0.004336	0.0314854	0.0358214
56	0.0047105	0.0298696	0.0345801
57	0.005121	0.0283366	0.0334576
59	0.0061025	0.0255026	0.0316051
60	0.0066995	0.0241938	0.0308933
62	0.0081715	0	0.0081715
64	0.0101265	0	0.0101265
65	0.011328	0	0.011328
66	0.012698	0	0.012698
70	0.0199575	0	0.0199575