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Low-Temperature Behavior of the Specific Heat for an *N*-Spin Ferromagnetic Material in an External Magnetic Field

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ABSTRACT

A ferromagnetic material in the absence of an external magnetic field shows the peak of its specific heat in low temperature, called the Schottky anomaly, which is vital in understanding the low-energy structure of a given material. A general formula for the low-temperature behavior of the specific heat of an N-spin ferromagnetic material in an external magnetic field (the generalized Schottky anomaly) is obtained for the first time. Also, as a representative example of ferromagnetic materials in an external magnetic field, the low-temperature behavior of the specific heat for the Ising ferromagnet is studied.

Keywords: *N*-spin ferromagnetic material, Generalized Schottky anomaly

Introduction

Most of materials are paramagnetic where microscopic (atomic or molecular) magnetic spins behave independently and a magnetic spin does not interact with any other magnetic spin [1, 2]. Consequently, a paramagnetic material does not show noticeable macroscopic magnetic properties. In an external magnetic field, microscopic spins in a paramagnetic material align along the direction of the external field, and the given paramagnetic material shows a noticeable macroscopic thermal property, called the Schottky anomaly [3]. It is a peculiar peak of the specific heat of the given material in low temperature, and it is vital in understanding the low- energy structure (in particular, the ground states and low-energy excited states) of a new material [3].

In modern industrial society, ferromagnetic materials [1, 2, 4] have been most widely used. In a ferromagnetic material, a microscopic magnetic spin interacts with neighboring magnetic spins, resulting in macroscopic magnetic force (that is, the force of a permanent magnet), even in the absence of an external magnetic field. In low temperature, a ferromagnetic material shows the Schottky anomaly [3] in no magnetic field. The low-temperature behavior of the specific heat for an *N*-spin ferromagnetic material has been understood well in the absence of an external magnetic field [5].

Recently, the generalized Schottky anomaly for the ferromagnetic materials in an external magnetic field has attracted experimental [6] and theoretical [7] studies. The generalized Schottky anomaly includes and combines both the Schottky anomaly of a paramagnetic material in an external magnetic field and the Schottky anomaly of a ferromagnetic material in no magnetic field. In this work, we study the low-temperature behavior (that is, the generalized Schottky anomaly) of the specific heat for an N-spin ferromagnetic material in an external magnetic field. In particular, for the first time, we derive a general formula for the low-temperature behavior of the specific heat of an *N*-spin ferromagnetic material in an external magnetic field. Furthermore, as a representative example of ferromagnetic materials, we discuss the low-temperature behavior of the specific heat for the Ising ferromagnet in an external magnetic field.

Low-Temperature Behavior of the Specific Heat in an External Magnetic Field

The grand partition function [8] $Z(T,B)$ of a magnetic material in an external magnetic field B for any temperature T is generally expressed as

$$Z(T,B) = \sum_E \sum_M g(E,M)\,\mathrm{Exp}[-\beta(E - BM)], \qquad (1)$$

where $g(E,M)$ is the density of states for a given energy E and magnetization M, and $\beta = 1/kT$ with k being the Boltzmann constant. Important thermodynamic functions such as internal energy, entropy, specific heat, spontaneous magnetization, and magnetic susceptibility are easily generated from the grand partition function. That is, if we get the grand partition function $Z(T,B)$ as a function of temperature T and magnetic field B, we are able to understand the properties of a given material.

In the low-temperature limit ($T \rightarrow 0$), the grand partition function in an external magnetic field B can be written as

$$Z(T,B) = g(E_0, M_0)\,\mathrm{Exp}[-\beta(E_0 - BM_0)] + g(E_0 + \Delta E, M_0 - \Delta M)\,\mathrm{Exp}[-\beta(E_0 - BM_0 + \Delta E + B\Delta M)], \qquad (2)$$

where E_0 is the energy of the ground state (that is, the lowest energy) and M_0 is the saturated magnetization (that is, the maximum magnetization). Here, ΔE is the difference between the ground-state energy and the first-existed-state energy. Similarly, ΔM is the difference between the ground-state magnetization and the maximum of possible magnetization values of the first excited states.

Next, from a differentiation of the logarithmic grand partition function [9], we can obtain the internal energy *U(T,B)* of a given magnetic material as

$$U(T,B) = -\frac{\partial}{\partial \beta}\ln Z = E_0 - BM_0 + \frac{(\Delta E + B\Delta M) g(E_0+\Delta E, M_0-\Delta M) e^{-\beta(E_0 - BM_0 + \Delta E + B\Delta M)}}{g(E_0, M_0) e^{-\beta(E_0-BM_0)} + g(E_0+\Delta E, M_0-\Delta M) e^{-\beta(E_0 - BM_0 + \Delta E + B\Delta M)}}. \tag{3}$$

If we define the ratio Ω of the density of the first excited state with the maximum magnetization to the density of the ground state as follows:

$$\Omega = \frac{g(E_0+\Delta E, M_0-\Delta M)}{g(E_0, M_0)}, \tag{4}$$

we can write the internal energy as

$$U = E_0 - BM_0 + \frac{(\Delta E + B\Delta M)\Omega \,\mathrm{Exp}[-\beta(\Delta E + B\Delta M)]}{1+\Omega\,\mathrm{Exp}[-\beta(\Delta E + B\Delta M)]}. \tag{5}$$

Now, the internal energy can be concisely written as

$$U(T,B) = E_0 - BM_0 + \frac{\Omega(\Delta E + B\Delta M)}{\Omega + \mathrm{Exp}[\beta(\Delta E + B\Delta M)]}. \tag{6}$$

Finally, from a differentiation of the internal energy [9], we can reach the specific heat *C(T,B)* of a given magnetic material as

$$C(T,B) = -\frac{1}{kT^2}\frac{\partial U}{\partial \beta} = \frac{(\Delta E + B\Delta M)^2}{kT^2}\frac{\Omega\,\mathrm{Exp}[\beta(\Delta E + B\Delta M)]}{(\Omega + \mathrm{Exp}[\beta(\Delta E + B\Delta M)])^2}. \tag{7}$$

The derived equation (7) is a general formula for the low-temperature behavior of the specific heat for an *N*-spin ferromagnetic material in an external magnetic field. This formula can be generally used to investigate and understand the low-temperature behavior of the specific heat of any magnetic material in an external magnetic field.

Ising Ferromagnet in an External Magnetic Field

As a representative example of ferromagnetic materials in an external magnetic field, we study the Ising ferromagnet which has played a central role in establishing modern theory of phase transitions and critical phenomena [10]. The Ising ferromagnet in an external magnetic field *B* on a lattice with *Ns* microscopic magnetic spins on lattice sites (one magnetic spin a site) and *Nb* bonds between two nearest-neighboring sites is defined by the following Hamiltonian

$$H = -J\sum_{<i,j>} S_i S_j - B\sum_i S_i, \tag{8}$$

where *J* is the exchange coupling constant ($J > 0$) between two nearest-neighboring magnetic spins S_i and S_j. Each magnetic spin S_i can take +1 (upward direction) or –1 (downward direction). The first large sigma is the sum over all possible nearest-neighboring bonds and the second large sigma is the sum over all lattice sites.

Now, we can define the unit-less energy

$$E = -\sum\nolimits_{<i,j>} S_i S_j \tag{9}$$

and the unit-less magnetization

$$M = \sum\nolimits_i S_i. \tag{10}$$

Then, the Hamiltonian of the Ising ferromagnet is simply written as

$$H(E,M) = JE - BM. \tag{11}$$

Because we conveniently set J = 1, the grand partition function

$$Z(T,B)=\sum_{E}\sum_{M}g(E,M)\,\mathrm{Exp}[-\beta H(E,M)] \tag{12}$$

of the Ising ferromagnet in an external magnetic field is the same as equation (1).

For the Ising ferromagnet, the density of ground states is two. One is the ground state with the maximum magnetization M_0, that is,

$$g(E=E_0=-Nb, M=M_0=Ns)=1. \tag{13}$$

The other is the ground state with the minimum magnetization $-M_0$, that is,

$$g(E=E_0=-Nb, M=-M_0=-Ns)=1. \tag{14}$$

If we consider the one-dimensional Ising ferromagnet with periodic boundary condition ($Ns = Nb = N$) in an external magnetic field, the density of the first excited states with the largest magnetization is

$$g(E=E_0+\Delta E, M=M_0-\Delta M)=Ns=N. \tag{15}$$

Here, the energy difference is $\Delta E = 4$ and the magnetization difference is $\Delta M = 2$. Because the ratio of the densities of the lowest-energy states is $\Omega = N$, we obtain finally the equation for the low-temperature behavior of the specific heat of the one-dimensional Ising ferromagnet in an external magnetic field as follows:

$$C(T,B)=\frac{(4+2B)^2}{kT^2}\frac{N\,\mathrm{Exp}[\beta(4+2B)]}{(N+\mathrm{Exp}[\beta(4+2B)])^2}. \tag{16}$$

This kind of equation for the generalized Schottky anomaly can be easily obtained for other ferromagnetic materials in an external magnetic field by using the equation (7).

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